Unit – I Electrostatics

1. Electrostatics:-

The branch of physics which deals with the study of charges at rest & the forces & fields, potential of charges is called electrostatics.

* Electrostatics was discovered around 600Bc by a Greek philosopher **Thales of Miletus**. He showed that when amber was rubbed with a cloth, then cloths starts attracting small piece of paper. In Greek amber is called electron so phenomenon was called electricity. (*Amber is a yellow gum like substance obtained from old plants).*

* In 1544 – 1603 **Sir William Gilbert** found that a force was present after rubbing of amber or any other substance called electrostatic force.

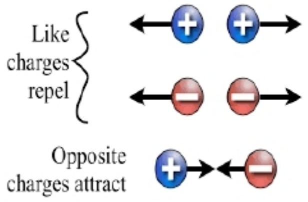
* In 1731 **Stephen Gray** found that charge can be moved through a metal for a long distance but not through a thread. This leads to two types of materials (i) Conductor (ii) Insulator

* In 1733 **Du Fay** discovered that charges are of two types + ve charge and – ve charge.

* In 1750 **Benjamin Franklin** proposed one fluid theory. He believed that charge occurs due to transfer of electrons. The excess electrons means -vely charged body & deficiency of electrons means + vely charged body.

2. What is electric charge:-

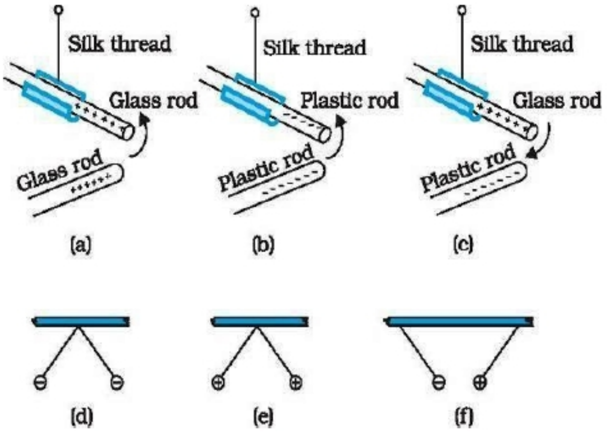
The property of electrons due to which they experience force of interactions is called electric charge.



- ✓ *The electrostatic force is much larger than gravitational force.*
- ✓ The charge on an electron is -1.6×10^{-19}C &
- ✓ Charge on a proton is=+ 1.6×10^{-19}C.
- ✓ Neutrons have no charge.

3. Two kinds of charge:-

In 1733 Du Fay discovered that charges are of two types. This can be shown by the following simple experiments.



Exp-1.

When two glass rods rubbed with silk are brought near each other then they started repel each other.

Exp.2

When two ebonite rods rubbed with cat's fur are also brought near each other, then they also repel each other.

Exp.3:-

But when a glass rod & ebonite rod are brought near each other then they started attracting each other.

- The property which differentiates the two kinds of charges is called polarity of charges.
- The charge on glass rod is called **vitreous charge** (Latin vitrum = glass) & the charge on amber when rubbed with wool is called **resinous charge** (Amber is resin)

Now according to Benjamin frankly, charges may define as

1. **Positive charge**:-

The charge developed on glass rod when rubbed with silk is called + ve charge. Or if numbers of p are greater than number of e^- in a body, then body is called +vely charged.

2. **Negative charge**:-

The charge developed on plastic rod when rubbed with wool is called negative charge. Or if number of e^- is more than number of p^+ in a body, and then body is called negative charged.

E, g:-Two kind of charge developed on rubbing

Column – I (+ ve charge)	Column – II (- ve charge)
Glass rod, fur or cat skin, woolen cloth	Silk cloth ebonite rod Amber rod plastic, rubber

4. Conductor, insulator & dielectrics:-

Conductor:-

A substance which can be used to conduct charge from one place to another place is called conductor. Silver is the best conductor. Other examples are copper, aluminum, iron mercury, earth, human body etc.

Insulator: -

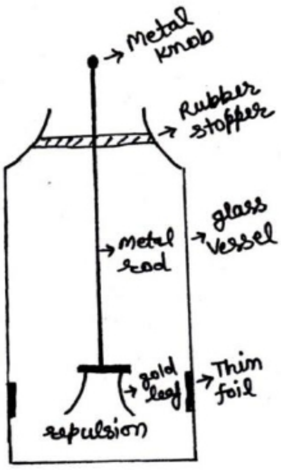
A substance which cannot be used to conduct electric charge is called insulator **e**, g: - glass, rubber, plastic, ebonite, mica, wax etc.

Dielectric:-

Dielectrics are that insulator which does not conduct electricity but on Appling external electric field charge induces on it e, g: - glass rod & paper acquire charge on rubbing.

5. Gold leaf electroscope (G L E)

It is a device which is used for detecting an electric Charge &identifying its polarity. It is consist of a vertical conducting rod passing through a rubber stopper fitted in the mouth of glass vessel. Two thin gold leafs are attached to the lower end of the rod. When a charged object touches the metal knob at the outer end of the rod, the charge flow down throw the leaves. The leaves diverge (moves away) due to repulsion of the like charge. The degree of divergence of the leaves gives measure of the amount of charge.



6. Method of Charging ^{imp}

(i) Charging by friction:-

When a body having loosely bounded electrons, is rubbed with a body having strongly bounded electrons, then both the bodies becomes charged by transfer of e^-.

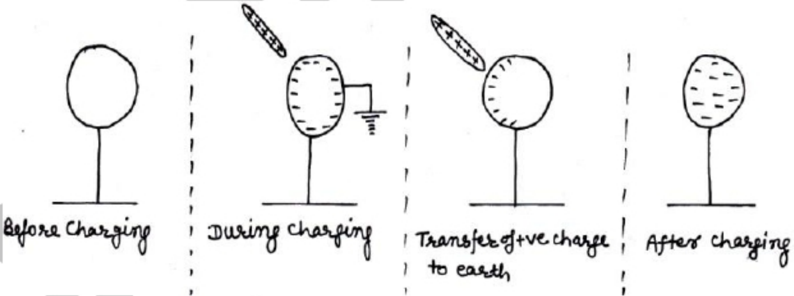
E,g:- when glass rod is rubbed with silk then glass rod become +vely charged by transferring electron & silk become -vely charged by acquiring electron.

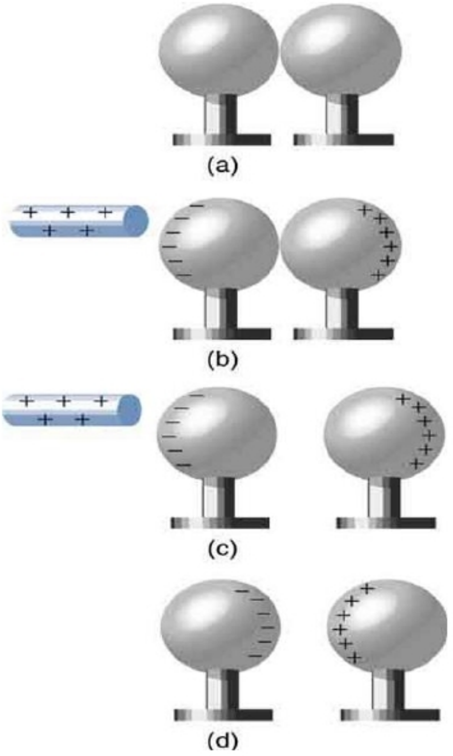
- *By loosing electron mass of the body decreases & by gaining e^- mass of body increases. As electron have mass 9.1×10^{-32}kg.*

(ii) Charging by electrostatic induction:-

The phenomenon of charging a neutral body by placing it in the neighboring of a charge body is called electrostatic induction.

When a positively charge body is bring toward a neutral body then charge separation takes place in neutral body which remains till the + vely charged body remains near to the neutral body. These induced charges may be explained in following ways.





(a) Charging by induction (by earthing a conductor);-

Suppose a neutral ball on an insulating stand & a positively charged glass rod is bringing toward it. Due to + ve charge on glass rod, the –ve charge induce toward the glass rod in the ball &+ ve charge induce on opposite side of the ball. By earthing the + ve charge of the ball, we get – vely charged ball. Hence charges can by induced by earthing.

(b) Charging by induction (By separating conductors)

Suppose two metallic balls mounted on insulating stands are placed in contact with each other. When a + vely charged glass rod is bring toward the balls then the face of first ball toward rod become – ve& face of second ball away from the rod becomes + vely charged. After separating balls, the ball A acquire -ve charge & ball B acquire +ve charge as shown in fig.

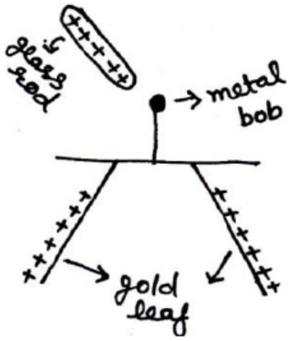
Q.1 how can you charge a metal sphere positively without touching it?

Ans: By bringing a negatively charged body towared metal sphere.

(iii) Charging by conduction (contact):-

Charging by conduction requires the actual contact between the two bodies.

In case of gold leaf electroscope when glass rod rubbed with silk is touched to the knob of leaf, than leaf diverse from their actual position, which remains separated even after removal of glass rod. Thus charging may be done by conduction.



7. Polar & Non Polar Bodies:-

Polar body:-

A body having different center of +ve& -ve charge is called polar body.

E, g: - HCl , H_2O etc.

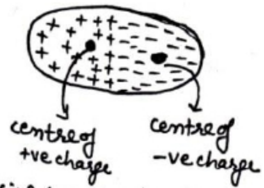


fig Polar body

Non polar body:-

A body having same center of +ve& -ve charge is called non polar body.

E, g: - H_2, O_2, N_2, etc

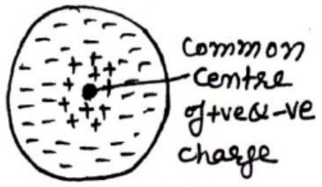
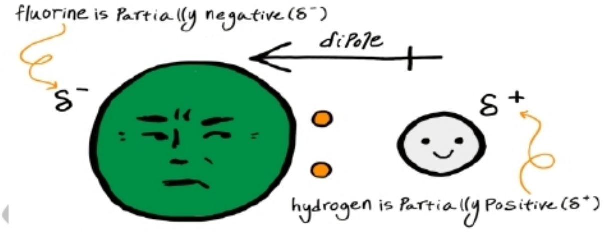


fig: Non Polar body



Q.2. (a) A comb run through one's dry hair attracts small bits of paper. Why? What happens if the hair is wet or if it is a rainy day? (Remember, a paper does not conduct electricity.)
Solution (a) this is because the comb gets charged by friction. The molecules in the paper get polarized by the charged comb, resulting in a net force of attraction. If the hair is wet, or if it is rainy day, friction between hair and the comb reduces. The comb does not get charged and thus it will not attract small bits of paper.

(b)Ordinary rubber is an insulator. But special rubber tyres of aircraft are made slightly conducting. Why is this necessary?
Solution: to enable them to conduct charge (produced by friction) to the ground; as too much of static electricity accumulated may result in spark and result in fire.

(c) Vehicles carrying inflammable materials usually have metallic ropes touching the ground during motion. Why?
Solution: Reason similar to (b)

(d) A bird perches on a bare high power line, and nothing happens to the bird. A man standing on the ground touches the same line and gets a fatal shock. Why?
Solution: Current passes only when there is difference in potential.

8. Some Basic Properties & Electric Charge:- imp

Same as mass, charge is also a fundamental & intrinsic property of matter. As like charges repel each other & unlike charge attract each other. There are three basic properties of charge.

(i) Quantization Of Charge:- (Discrete Nature of charge)

According to quantization nature of charge, the charge on a body is always whole number multiple of charge on an electron.

I,e charge on a body $q = \pm ne$ n= 1, 2, 3, 4, ………but n≠ ½, 3/2, 1.5 etc.

Where e = 1.6×10^{-19}C (charge on an electron) & n is a whole number

- *During rubbing, electron can transfer from one body to another body in a whole number. So charge is quantized as half electron cannot be transferred.*
- *An elementary particle quart of very small life time have fraction of charge.*
  $\pm\frac{2}{3}e$ *&* $\pm\frac{1}{3}e$*. (it is exception of quantization of charge)*

Q.3. If 10^9 electrons move out of a body to another body every second, how much time is required to get a total charge of 1 C on the other body?

Solution: Therefore the charge given out in one second is= $1.6 \times 10^{-19} \times 10^9$ C = 1.6×10^{-10} C.

The time required to accumulate a charge of 1 C is $t = \frac{1C}{1.6 \times 10{-}10\ C/s}$= 6.25×10^9 s = $\frac{6.25 \times 10^9}{365 \times 24 \times 3600}$ years = 198 years.

Thus to collect a charge of one coulomb, from a body from which 10^9 electrons move out every second, we will need approximately 200 years.

✓ One coulomb is, therefore, a very large unit for many practical purposes.

Q.4. how much positive and negative charge is there in a cup of water?

Solution: Let us assume that the mass of one cup of water is 250 g.

The molecular mass of water is 18g. so we can say 18g of water contain $6.022\text{x}10^{23}$ molecules

1g of water contain =$\frac{6.02 \times 10\,23}{18}$ molecules.

Therefore the number of molecules in one cup of water is $= \frac{250}{18} \times 6.02 \times 10^{23}$.

Each molecule of water contains two hydrogen atoms and one oxygen atom, i.e., 10 electrons and 10 protons.

Hence the total positive or total negative charge = $\frac{250}{18} \times 6.02 \times 10^{23} \times 10 \times 1.6 \times 10^{-19}$ C = 1.34×10^7 C.

Q.5. A polythene piece rubbed with wool is found to have a negative charge of 3×10^{-7} C. (a) Estimate the number of electrons transferred (from which to which?) (b) Is there a transfer of mass from wool to polythene?

(a) When polythene is rubbed against wool, electrons get transferred from wool to polythene. Hence, wool becomes positively charged and polythene becomes negatively charged.

Amount of charge on the polythene piece, q = -3×10^{-7} C
Amount of charge on an electron, e = -1.6×10^{-19} C
Number of electrons transferred from wool to polythene = n
As q = ne Therefore, the number of electrons transferred from wool to polythene is 1.87×10^{12}.

(b) Yes. There is a transfer of mass taking place. This is because an electron has mass, me = 9.1×10^{-3} kg.
Total mass transferred to polythene from wool, m = $m_e \times n = 9.1 \times 10^{-31} \times 1.85 \times 10^{12} = 1.706 \times 10^{-18}$ kg.
Hence, a negligible amount of mass is transferred from wool to polythene.

(ii) Conservative nature of electric charge:-

According to law of conservation of electric charge, the total charge on an isolated system remains constant; it can neither be created nor be destroyed& can only be transferred from one body to another body. E,g:-

(i) $\text{NaCl} \rightarrow \text{Na}^+ + \text{Cl}^-$

Here charge on reactant is zero & charge on product = +1-1=0

So law of conservation of charge hold good.

(ii) γ ray $\rightarrow$ (e^-) electron $+ e^+$ (positron)

(Zero charge) $\longrightarrow$ (-1) + (+1) = zero charge

(iii) Addition nature of electric charge:-

According to additive nature of charge, the net charge on an isolated system can be simply obtained by adding all the charges scalarly.

If a system having charge $q_1, q_2, q_3 \ldots q_n$. Then total charge on the system $q = q_1 + q_2 + q_3 \ldots\ldots\ldots q_n$.

E.g. If system have four charges 2μC, 3μC, 4μC, -5μC

Then total charge q = 2μC + 3 μC + 4 μC + (-5 μC) = 4 μC

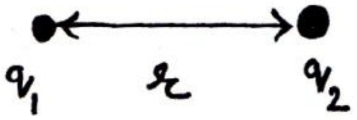
9. comparison of charge & mass :-

	Electric Charge	Mass
1	Electric charge may be + ve, -ve or zero.	The mass of a body is always a positive.
2	Electric charge is always quantized.	Quantization of mass is yet not obeyed.
3	Charge on a body does not depend on its speed.	Mass of the body increase with its velocity.
4	Charge is strictly conserved.	Mass is not conserved as it may convert into energy.
5	The electrostatic force between two charges may be attractive or repulsive.	Gravitational forces between two masses are always attractive.
6	Electrostatic force between different charges may cancel out.	Gravitational forces between different masses never cancel out.
7	A charged body always has some mass.	A body having mass may not have any charge.

10. Coulombs Law In Electrostatics :-(Scalar Form) $^{\text{M.Imp}}$

In 1785, the French physicist Charles Augustan coulomb measured the electric force between small charged spheres by using a torsion balance & gave a law which is as below.

Coulombs law states that the force of interaction between two stationary charge is directly proportion to the product of magnitude of charges & inversely proportional to the square of distance between the charges. This force acts along the line joining among the charges.



I, e $F \propto q_1 q_2$ ………… 1

$F \propto \frac{1}{r^2}$ …………….. .2

Combining equations 1& 2 we get

$$F \propto \frac{q_1 q_2}{r^2}$$

Or $$F = k\frac{q_1 q_2}{r^2}$$

Where k is constant of proportionality & its value is $k = \frac{1}{4\pi\varepsilon_0} = 9\times10^9\ \text{Nm}^2\text{C}^{-2}$

here ε_o is a another constant called permittivity of free space & its value is $\varepsilon_o = 8.854\times10^{-12}\text{C}^2\text{N}^{-1}\text{m}^{-2}$

So coulomb's law becomes

$$\mathbf{F= \frac{kq_1q_2}{r^2} = \frac{1}{4\pi\,\varepsilon_o}\frac{q_1q_2}{r^2} = 9\times10^9\frac{q_1q_2}{r^2}}$$

Unite of charge:-

The S.I unite of electric charge is 1 coulomb.

As we know $F = 9\times10^9\ \frac{q_1q_2}{r^2}$

If $F = 9\times10^9\text{N},\ q_1 = q_2 = q$ (say), and $r = 1m$

Then by Coulombs law $9\times10^9 = 9\times10^9\ q^2$

$\Rightarrow$ $q^2 = 1C$ Or $q = \pm 1C$

Hence if two equal charge repels each other with a force of 9×10^9N when placed one meter distance apart. Then charge is said to be one coulomb.

- The c, g, s unit of charge is Stat Coulomb (stat C) or electrostatic unit of charge (emu)

One stat coulomb is that charge which repel an identical charge in vacuum at distance of 1 cm with a force of 1 dyne **1 Coulomb =3× 10^9 statCoulomb (e m u)**

- In electromagnetic c, g, s unit of charge is abCoulomb's or electromagnetic unit of charge (e m u)

$$1 \text{ coulomb} = \frac{1}{10}\text{ abCoulomb's} = \frac{1}{10}\text{ emu of charge.}$$

Significance of coulombs law:-

(i) It tells us about the force which bounds the electrons around the nucleus to form a atom.

(ii) It tells us about the force which binds the molecule to form solids & liquids

Limitations of coulomb's law:-

(i) Coulombs law is applicable only on point charges.

(ii) It holds good only when charges are in rest only.

11. Comparison between coulomb's force &Newton's gravitational force:-

Similarity

(i) Both obey inverse square law.

(ii) Both forces are central forces.

(iii) Both forces are conservative forces.

(iv) Both forces are directly proportional to product of interacting Patrick

Dissimilarity:-

(i) Coulomb's force is attractive as well as repulsive while Gravitational force is always attractive in nature.

(ii) Coulomb's force is much stronger than gravitational force (10^{36}times)

(ii) Coulombs force depends upon the medium in which charges are placed while gravitational force does not depends upon medium.

- **Q.6 How much stronger is coulombs force from gravitational force between a electron & proton separated by r distance?**

I.e. $F_e = \frac{9\times10^9 e^2}{r^2}$ and $F_m = \frac{6.67\times10^{-11} m_e{}^2 m_p{}^2}{r^2}$

$$\Rightarrow \quad \frac{Fe}{Fm} = \frac{9\times10^9\times(1.6\times10^{-19}).^2}{6.6\times10^{-11}\times(9.1\times10^{-31}\times1.6\times10^{-27}}$$

Or $\frac{Fe}{FG} = 1.27\times 10^{36}$

Hence Fe is 10^{36} time stronger then F_G

- **Q.7 : What is the force between two small charged spheres having charges of 2×10^{-7} C and 3×10^{-7} C placed 30 cm apart in air?**

  Ans: $F= 9\times10^9 \frac{q_1q_2}{r^2} = F = 9 \times 10^9 \frac{2\times10^{-7}\times3\times10^{-7}}{(30\times10^{-2})^2}$ $=6\text{x}10^{-3}$N

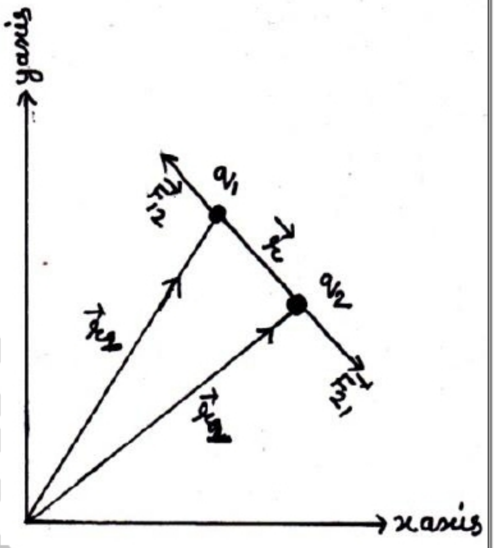
- **Q.8: The electrostatic force on a small sphere of charge 0.4 μC due to another small sphere of charge – 0.8 μC in air is 0.2 N. (a) what is the distance between the two spheres? (b) What is the force on the second sphere due to the first?**

  Ans: (a) $r = \sqrt{144 \times 10^{-4}} = 0.12m$

  (b) both have equal force so F=0.2N

12. Coulomb's Law in vector from:- imp

Suppose two charge q_1& q_2 placed at A & B having position vectors $\overrightarrow{r_1}$ & $\overrightarrow{r_2}$ from origin O. Now again suppose that charge q_1 exert $\vec{F}_{21}$ force on q_2& q_2 exerts $\vec{F}_{12}$ force on q_1.



Then from coulomb's Law

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{|\vec{r}_{21}|^2}.\hat{r}_{21} \quad \ldots\ldots\ldots\ldots\ldots\ldots 1$$

Where $\vec{r}_{21}$ is a position vector acting from q_2 to q_1.

Again from Coulomb's Law force on q_2 to q_1

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{|\vec{r}_{12}|^2}.\ \hat{r}_{12} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$$

Where $\hat{r}_{12}$is a position vector, which acts along q_1 from q_2.

Clearly $\vec{r}_{12}$=- $\vec{r}_{21}$

So from eq^n 2 we get

$$\vec{F}_{21} = -\frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{|\vec{r}_{21}|^2}\hat{r}_{21} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3$$

Compairing eq^n 1 & 3 we get

$$\vec{F}_{12} = -\vec{F}_{21}$$

Hence Coulomb's Law in vector from obey Newton's third Law of motion.

Also from diagram $\vec{r}_{21} = \vec{r}_1 - \vec{r}_2$ (from Δ law of vector addition)

& $\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$ (from △ law of vector addition)

So eq^n 1 & 2 becomes

$$\vec{F}_{12} = \frac{1}{4\pi\,\varepsilon_o}\frac{q_1q_2}{|\vec{r}_1-\vec{r}_2|^2}\frac{(\vec{r}_1-\vec{r}_2)}{|\vec{r}_1-\vec{r}_2|}\ (\because \hat{r}=\frac{\vec{r}}{\overrightarrow{|r|}})$$

Or

$$\vec{F}_{12} = \frac{1}{4\pi\,\varepsilon_o}\frac{q_1q_2}{|\vec{r}_1-\vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2) \quad \ldots\ldots\ldots\ldots\ldots\ldots 4$$

& similarly

$$\vec{F}_{21} = \frac{1}{4\pi\,\varepsilon_o}\frac{q_1q_2}{|\vec{r}_2-\vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1) \quad \ldots\ldots\ldots\ldots\ldots\ldots 5$$

Equation 4 and 5 represents **Coulombs Law in Position Vector Form**.

- **Q .9: Four point charges q_A = 2 μC, q_B = –5 μC, q_C = 2 μC, and q_D = –5 μC are located at the corners of a square ABCD of side 10 cm. What is the force on a charge of 1 μC placed at the centre of the square?**
  Ans: 0
- **Q.10. (a) Two small insulated charged copper spheres A and B have their centers separated by a distance of 50 cm. What is the mutual force of electrostatic repulsion if the charge on each is 6.5 × 10^{-7} C?**
- **(b) What is the force of repulsion if each sphere is charged double the above amount, and the distance between them is halved?**
  Ans: (a) force between the two spheres is 1.52 × 10^{-2} N. (b) 0.243 N.

11. Three equal charges, $2\cdot 0 \times 10^{-6} C$ each, are held fixed at the three corners of an equilateral triangle of side 5 cm. Find the Coulomb force experienced by one of the charges due to the rest two.

Ans 24.9 N at 30° with the extended sides from the charge under consideration

12. Four equal charges $2\cdot 0 \times 10^{-6} C$ each are fixed at thefour corners of a square of side 5 cm. find the Coulomb force experienced by one of the charges due to the rest three.

Ans 27.5 N at 45° with the extended sides of the square from the charge under consideration

13. Dielectric Constant or Relative Permittivity:- M.Imp

Permittivity is a property of the medium which determines the electric force between the charges situated in that medium.

Relative permittivity:-

As we know coulomb's force between the two charge placed in vacuum is

$$F_o = \frac{1}{4\pi\,\varepsilon_o}\frac{q_1 q_2}{r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 1$$

Where ε_o is the permittivity in free space

Again force between the same charges, when placed in a medium is

$$F_m = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 2$$

Here ε is the permitivity in the medium

Dividing eq^n 1 & 2 we get $\frac{F_o}{F_m} = \frac{\frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{r^2}}{\frac{1}{4\pi\,\varepsilon}\frac{q_1q_2}{r^2}} = \frac{\frac{1}{\varepsilon_o}}{\frac{1}{\varepsilon}} = \frac{\varepsilon_o}{\varepsilon} = \varepsilon_r$ or K (note: $k = \frac{1}{4\pi\,\varepsilon_o}$ and $K = \varepsilon_r$)

Where ε_r is called relative permitivity.

Hence *relative permittivity may be defined as the ratio of force between two charges in vacuum to the force between the charges in medium.* Or

Relative permittivity may be defined as the ratio of permittivity in the medium to the permittivity in free space.

- ✓ Hence coulombs law for material medium becomes

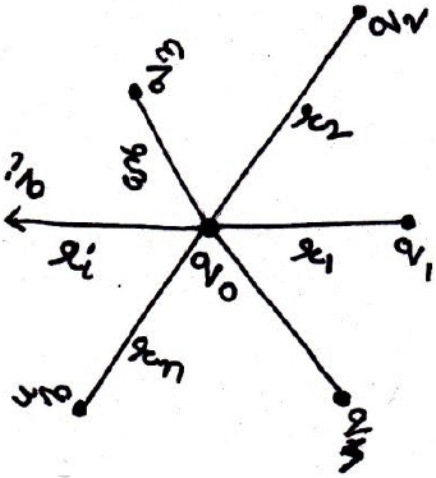
$$\mathbf{F_m = \frac{1}{4\pi\varepsilon\,\varepsilon_r}\frac{q_1 q_2}{r^2}}$$

Q.13. what will be the Coulombs force if two charges are placed in water?

Clearly $F_m = \frac{F_o}{\varepsilon_r}$ If charges are placed in water (ε_r=80) then force between charges becomes $F_w = \frac{F_o}{80}$.
Here we can see that force reduces by 80 in water as compare to vacuum

14. Force Between Multiple Charges: The superposition Principle:-

The principle of superposition states that if there are a number of charges exerting force on a single charge, then total force on single charge will be equal to sum of all the forces exerted by individual charge on the charge.



Suppose there are n charges q_1, q_2 , q_3…q_n exerting force on a charge q_0. Then from coulomb's law force on q_0 due to q_1

Is $$\vec{F}_{01} = \frac{1}{4\pi\ \varepsilon_o}\frac{q_0 q_1}{|\vec{r}_0-\vec{r}_1|^3}(\vec{r}_0-\vec{r}_1) \text{……………………………………} \quad 1$$

Again force on q_0 due to q_2 is

$$\vec{F}_{02} = \frac{1}{4\pi\varepsilon_o}\frac{q_0 q_2}{|\vec{r}_0-\vec{r}_2|^3}(\vec{r}_0-\vec{r}_2) \quad \text{………………..} \quad 2$$

Similarly

$$\vec{F}_{03} = \frac{1}{4\pi\varepsilon_o}\frac{q_0 q_3}{|\vec{r}_0-\vec{r}_3|^3}(\vec{r}_0-\vec{r}_3) \quad \text{………………………………} \quad 3$$

$$\vec{F}_{0n} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{|\vec{r}_0-\vec{r}_n|^3}(\vec{r}_0-\vec{r}_n) \quad \text{…………………………} \quad n$$

Adding all the eq^n s we get

$$F = \vec{F}_{01}+\vec{F}_{02}+\vec{F}_{03} \text{………} \vec{F}_{0n} = \frac{1}{4\pi\ \varepsilon_o}\frac{q_0 q_1}{|\vec{r}_0-\vec{r}_1|^3}(\vec{r}_0-\vec{r}_1) + \frac{1}{4\pi\varepsilon_o}\frac{q_0 q_2}{|\vec{r}_0-\vec{r}_2|^3}(\vec{r}_0-\vec{r}_2) \dots\dots \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{|\vec{r}_0-\vec{r}_n|^3}(\vec{r}_0-\vec{r}_n)$$

$$= \frac{q_0}{4\pi\varepsilon_o}\left(\frac{q_1}{[\vec{r}_0-\vec{r}_1]^3}(\vec{r}_0-\vec{r}_1) + \frac{q_2}{[\vec{r}_0-\vec{r}_2]^3}(\vec{r}_0-\vec{r}_2) \dots\dots \frac{q_n}{[\vec{r}_0-\vec{r}_n]^3}(\vec{r}_0-\vec{r}_n)\right)$$

So total force on a charge $$\vec{\boldsymbol{F}} = \frac{\boldsymbol{q_0}}{\boldsymbol{4\pi\ \varepsilon_o}}\sum_{i=0}^{n}\frac{\boldsymbol{q_i}}{\left|\vec{\boldsymbol{r}}_0-\vec{\boldsymbol{r}}_i\right|^3}(\vec{r}_0-\vec{r}_i)$$

- ❖ In scalar form this expression may be written as

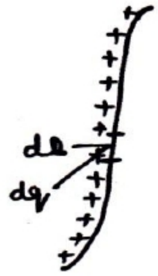
$$F = \frac{q_0}{4\pi\ \varepsilon_o}\sum_{i=0}^{n}\frac{q_i}{{r_i}^2}$$

- ❖ In vector form $$\vec{F} = \frac{q_0}{4\pi\ \varepsilon_o}\sum_{i=0}^{n}\frac{q_i}{\left|\vec{r}_i\right|^2}\hat{r}_i$$

15. Force on a point charge due to continuous charge distribution:-

A continuous charge distribution is a system of charge lying at infinitelly small distances from each other. There are three types of continuous charge distribution.

(a) Linear or line charge distribution:-



Ifcharges are arranged in such a way that they seem to be like a line then the distribution of charges is called line charge distribution.

The line charge density λ may be defined as the charge per unit length.

I, e $$\boldsymbol{\lambda} = \frac{\boldsymbol{charge}}{\boldsymbol{length}} = \frac{\boldsymbol{dq}}{\boldsymbol{dl}}$$

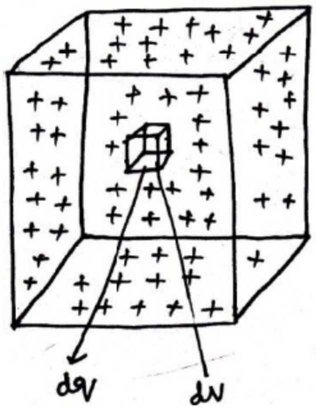
(b) Surface or area charge distribution:-

If charge are arranged in such a way that they seems to be a surface (plane) then the distribution of charge are called surface distribution of charge

The surface charge density σ may be defined as then charge per unit area of the conductor.

I, e $$\sigma = \frac{charge}{area} = \frac{dq}{ds}$$

(d) Volume Charge distribution:-



If charges are arranged in such a way that they seems to be like a volume, then the distribution of the charges is called *volume distribution of charge*.

The volume charge density ρ may be defined as the charge per unit volume of the conductor.

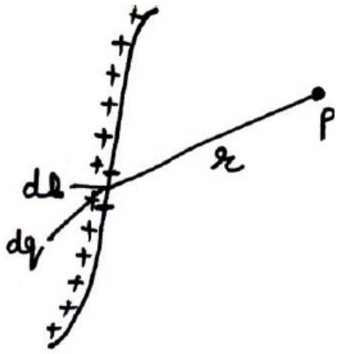
i,e $$\rho = \frac{charge}{volume} = \frac{dq}{dv}$$

16 force due to continuous distribution of charge:

(i) Forces at a point due to continuous line distribution of charges:-

Suppose a point P having r distance from a point O on continuous line distribution of charge. Than small force at charge $\boldsymbol{q_0}$placed at P due to small charge dq on small surface ds may be given by coulomb's law,

$$d\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_0 dq}{\left|\vec{r}\right|^2}\hat{r}$$



But dq = λdl where λ is line charge density.

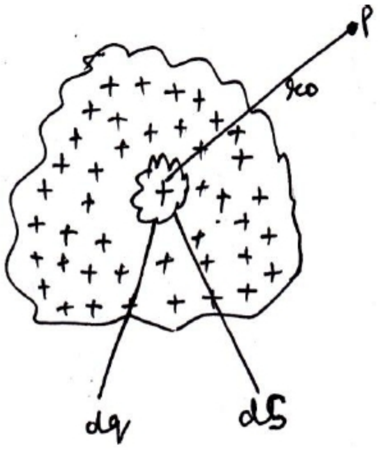
So $$d\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_0\lambda dl}{\left|\vec{r}\right|^2}\hat{r}$$

For complete length $$\vec{\boldsymbol{F}} = \frac{\mathbf{1}}{\mathbf{4\pi\varepsilon_0}}\int\frac{\boldsymbol{q_0\lambda dl}}{\left|\vec{r}\right|^2}\hat{\boldsymbol{r}}$$

(ii) Force due to continuous surface distribution of charge:-

Suppose a test charge q_0 is placed at point p having r distance from small surface ds having charge dq. Then force on q_0 is

$$d\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_0 dq}{\left|\vec{r}\right|^2}\hat{r}$$



Where surface charge density $\sigma = \frac{dq}{ds} \Rightarrow$ dq = σ ds

$$= d\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_0\sigma\, ds}{\left|\vec{r}\right|^2}\hat{r}$$

So the force at p due to complete surface is

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\int\frac{q_0\sigma\, ds}{\left|\vec{r}\right|^2}\hat{r}$$

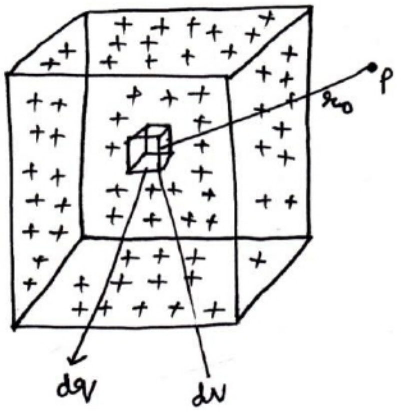
(iii) Force due to continuous volume distribution of charges.

Suppose a test charge q_0 is placed at point p having distance r from small volume dv having charge dq. Then force on q_0 due to dq $\quad d\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_0 dq}{\left|\vec{r}\right|^2}\hat{r}$

where volume charge density $\quad \rho = \frac{dq}{dv} \Rightarrow$ dq = ρdv

$$\Rightarrow \quad d\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_0\rho\, dv}{\left|\vec{r}\right|^2}\hat{r}$$

Hence force at p due to complete surface is $\vec{F} = \frac{1}{4\pi\varepsilon_0}\int\frac{q_0\rho\, dv}{\left|\vec{r}\right|^2}\hat{r}$



Electric Field

17. Electric Field:-

The space around a positive charge in which its force of interaction may be experienced is called electric field.

Electric field intensity:- M.Imp

The force experienced by a unit + ve charge in the field of another charge is called electric field intensity. It is a vector quantity & denoted by $\vec{E}$.

I,e $$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$$

Here the charge q_0 should be so small, that it does not disturb the position of original charge.

Also $$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_o}\frac{q_0 q}{\left|\vec{r}\right|^2 q_0}\hat{r} = \frac{1}{4\pi\varepsilon_o}\frac{q}{\left|\vec{r}\right|^2}\hat{r}$$

Or $$\mathbf{E} = \frac{\mathbf{1}}{\mathbf{4\pi\varepsilon_o}}\frac{\boldsymbol{q}}{\boldsymbol{r^2}}$$

Unit &Dimensions:-As $$E = \frac{F}{q} = \frac{MLT^{-2}}{AT} = [MLT^{-3}A^{-1}]$$

& unit of E = NC^{-1} or ***volt per meter***.

- *electric field is stronger near the charge & decreases exponentially with distance & becomes zero at ∞.*

Physical Significance Of Electric Field:- imp

By knowing electric field at any point, we can determine force on a charge placed at that point.

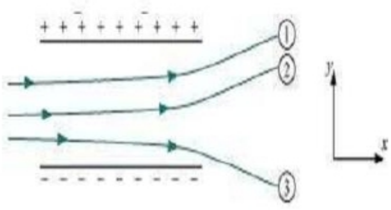
I.e. $Electric\ force = Charge \times Electric\ field$

Thus electric field experience a intermediate role between two charge

As $charge \rightleftharpoons electric\ field \rightleftharpoons charge$

It is a characteristic of the system of charges & is independent of the test Charge that we place to determine the field.

Q.14: shows tracks of three charged particles in a uniform electrostatic field. Give the signs of the three charges. Which particle has the highest charge to mass ratio?



Answer: Opposite charges attract each other and same charges repel each other. It can be observed that particles 1 and 2 both move towards the positively charged plate and repel away from the negatively charged plate. Hence, these two particles are negatively charged.

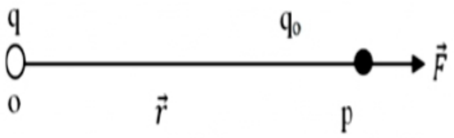
It can also be observed that particle 3 moves towards the negatively charged plate and repels away from the positively charged plate. Hence, particle 3 is positively charged.

The charge to mass ratio (emf) is directly proportional to the displacement or amount of deflection for a given velocity. Since the Deflection of particle 3 is the maximum, it has the highest charge to mass ratio.

18. Electric Field due to a point charge:-

Suppose a point charge $q_0$is placed at point p having r distance from a charge q placed at point o.

Then according to coulomb's law the force of interaction between



The charge is $F= \frac{1}{4\pi\varepsilon_o}\frac{qq_0}{r^2}\hat{r}$

& the electric field at point p is

$$\vec{E} = \lim_{q_0\to 0}\frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_o}\frac{q_0q}{\vec{r}^{\,2}q_0}\hat{r} = \frac{1}{4\pi\varepsilon_o}\frac{q}{\vec{r}^{\,2}}\hat{r}$$

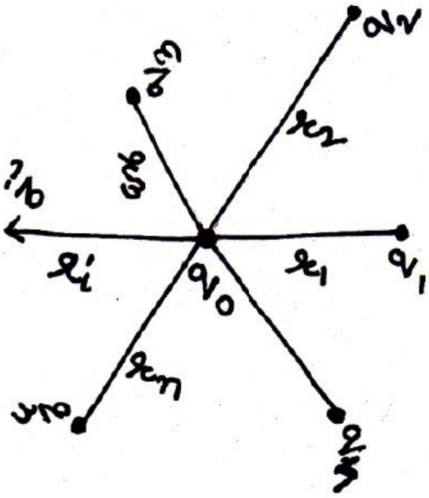
The magnitude of electric field is

$$\mathbf{E}= \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$$

* Clearly $E \propto \frac{1}{r^2}$ *, it mesas E is same in all direction & does not depends upon direction of r. Such a field is called* ***spherically symmetric or radial field.***

19. Electric field due to a system of point charges:-

Suppose q_0 charge is placed at point p having Distances $\vec{r}_1, \vec{r}_2$, $\vec{r}_3$ .... $\vec{r}_n$,. From charges q_1, q_2.......... q_n.. Respectively. Then total electric field at q_0 due to all charges may be calculated as given below.



Electric field at q_0 due to q_1 Charge is

$$\vec{E}_{01} = \frac{1}{4\pi\varepsilon_o}\frac{q_0q_1}{r^2}\hat{r}_{10}\text{.......................} \quad 1$$

Again electric field at q_0 due to q_2 charge is

$$\vec{E}_{02} = \frac{1}{4\pi\varepsilon_o}\frac{q_0q_1}{r^2}\hat{r}_{20}\text{.......................} \quad 2$$

& Electric field at q_0 due to q_3 charge is

$$\vec{E}_{03} = \frac{1}{4\pi\varepsilon_o}\frac{q_0q_1}{r^2}\hat{r}_{30}\text{.......................} \quad 3$$

Similarly electric field at q_0 due to q_n charge will be

$$\vec{E}_{0n} = \frac{1}{4\pi\varepsilon_o}\frac{q_0q_1}{r^2}\hat{r}_{n0}\text{.......................} \quad n$$

Now the total electric field at q_0 due to all charges will be

$$E=\vec{E}_{01} + \vec{E}_{02} + \vec{E}_{03} + \vec{E}_{04} + \cdots \vec{E}_{0n} = \frac{1}{4\pi\varepsilon_o}\frac{q_0q_1}{r^2}\hat{r}_{10} + \frac{1}{4\pi\varepsilon_o}\frac{q_0q_1}{r^2}\hat{r}_{20} + \text{............}\frac{1}{4\pi\varepsilon_o}\frac{q_0q_1}{r^2}\hat{r}_{n0}$$

Or $$E = \frac{q_0}{4\pi\varepsilon_o}\left(\frac{q_1}{r^2}\hat{r}_{10} + \frac{q_2}{r^2}\hat{r}_{20} + \frac{q_3}{r^2}\hat{r}_{30} \dots\dots\dots\dots\dots\dots\dots.\frac{q_n}{r^2}\hat{r}_{n0}\right)$$

Or $$\mathbf{E} = \frac{q_0}{4\pi\varepsilon_o}\sum_{i=0}^{n}\frac{q_i}{\left|\vec{r}_i\right|^2}\hat{r}_i$$

Q. 15: Two point charges $q_A = 3$ ⍰C and $q_B = -3$ ⍰C are located 20 cm apart in vacuum.
(a) What is the electric field at the midpoint O of the line AB joining the two charges?
(b) If a negative test charge of magnitude 1.5×10^{-9} C is placed at this point, what is the force experienced by the test charge?

Ans: **(a)** The electric field at mid-point O is 5.4×10^{6} N C^{-1} along OB.
(b) A test charge of amount 1.5×10^{-9} C is placed at mid-point O. $q = 1.5 \times 10^{-9}$ C
Force experienced by the test charge = $F = qE = 1.5 \times 10^{-9} \times 5.4 \times 10^{6} = 8.1 \times 10^{-3}$ N.
The force is directed along line OA.

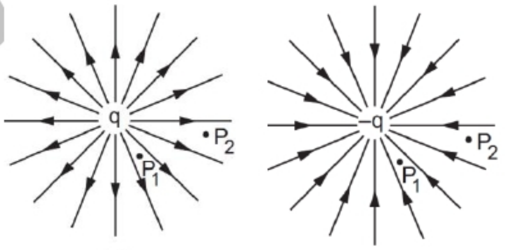
Question 16: An oil drop of 12 excess electrons is held stationary under a constant electric field of 2.55×10^{4} N C^{-1} in Millikan's oil drop experiment. The density of the oil is $1.26\ g\ cm^{-3}$. Estimate the radius of the drop. ($g = 9.81$ m s^{-2}; $e = 1.60 \times 10^{-19}$ C).

Ans: Excess electrons on an oil drop n = 12. $E = 2.55 \times 10^{4}$ N C^{-1}
And $\rho = 1.26$ gm/cm^3 = 1.26×10^{3} kg/m^3.g =9.81ms^{-2}. $e = 1.6 \times 10^{-19}$ C .Radius of the oil drop = r
Force (F) due to electric field E is equal to the weight of the oil drop F = W
$qE = mg$ Where, q = Net charge on the oil drop = ne
m = Mass of the oil drop = Volume of the oil drop × Density of oil = 9.82×10^{-4} kg
Therefore, the radius of the oil drop is $r = \sqrt[3]{\frac{3Ene}{4\pi\rho g}} = 9.82 \times 10^{-4}$ mm.

20. Electric field lines:- M.Imp

The electric field lines are the path followed by a unit positive charge when placed in the field of a charge. These lines are imaginary lines & tangent drawn on these lines gives the direction of electric field at that point.

Properties of electric field lines:-

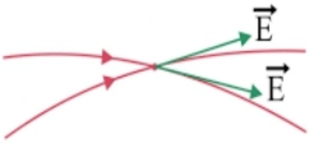


1. The electric lines of forces are continuous curves having no any breakages.

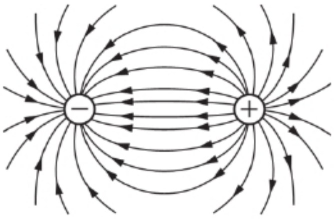
2. These lines originating from a +ve charge in Outward direction & originating from +ve charge is in inward direction.

3. The tangent drawn on an electric line of force gives the direction of electric field at that point.

4. No two electric lines of forces can intersect each other. Because if they do so, it means there are two direction of a single field at same point. This is not possible.

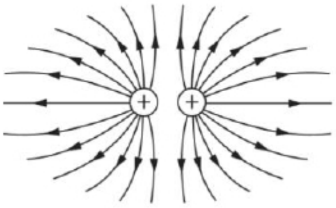


5. The electric field lines are always normal to the surface from which they originate or end.



6. Larger the field lines in a region means more is electric field in that region & vice versa.

7. These lines do not pass through a conductor because electric field inside the conductor is always zero.



8. Electric field lines emits from a +ve charge & ends at –ve charge.

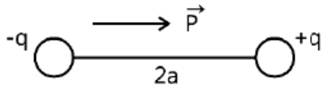
9. Electric lines of forces of two positive charges repel each other.

21. Electric dipole:- $^{\text{M.Imp}}$

An electric dipole is consist of two equal & opposite Charge –q & +q separated by a small distance 2a.

Electric dipole moment:-

The product of either charge of electric dipole & the distance between the charges is called electric dipole moment. It is denoted by $\vec{P}$.



I.e. $\vec{P} = \pm q\,.\,\overrightarrow{2a}$

- ✓ *Electric dipole moment is a vector quantity having direction from* ***–ve to +ve.***

Unit &Dimensional formula:-

(i) The S.I unit of electric dipole is Coulomb metre (Cm)

(ii) The practical unit of electric dipole is Debye (D)

$$1\text{D} = 10^{-10}\,\text{Franklin} \times 1\text{A}^\circ$$

$$= 10^{-10} \times \frac{1}{3} \times 10^{-9} \times 10^{-10} \qquad (\because 1\text{C} = 3 \times 10^{9}\,\text{Fr})$$

Or $1\text{D} = 3.33 \times 10^{-30}\,\text{Cm}$

Dimensional formula: - As P= q.2a = [AT] [L] = [$M^0 L$ TA]

- ✓ <u>An **ideal dipole** has very small distance between the large charges.</u>
- ✓ An electric field produced by an electric dipole is called dipole is called **dipole field.**

Physical Significance of Electric dipole:

An electric dipole has a common existence in Nature. A molecule consist of a +ve& -ve charge is an electric dipole. It is used in study of effect of electric field on insulators & in study of radiations of energy from an antenna.

Question: 17 Two particles with equal charge magnitudes $2.0 \times 10^{-7} C$ but opposite in signs are held 15 cm apart. What are the magnitude and direction of the Electric Field E at the point midway between the charges?

Ans $E_{net} = 2E_1$ **here** $E_1 = \frac{k.2.0 \times 10^{-7} C}{\left(\frac{15}{2} \times 10^{-2} m\right)^2} = 3.2 \times 10^5 \frac{N}{C} \Rightarrow E_{net} = 6.4 \times 10^5\ N/C$ **toward left**

Question 18: A system has two charges $q_A = 2.5 \times 10^{-7}\ C$ and $q_B = -2.5 \times 10^{-7}$ C located at points A: (0, 0, – 15 cm) and B: (0, 0, + 15 cm), respectively. What are the total charge and electric dipole moment of the system?

Answer: Distance between two charges at points A and B, $d = 15 + 15 = 30\ cm = 0.3\ m$
Electric dipole moment of the system is given by, $P = qA \times d = qB \times d = 2.5 \times 10^{-7} \times 0.3 = 7.5 \times 10^{-8}\ Cm$ along positive z-axis.
Therefore, the electric dipole moment of the system is $7.5 \times 10^{-8}\ Cm$ along positive z–axis.

22. Electric Field at Arial Point of an electric dipole:-^{imp}

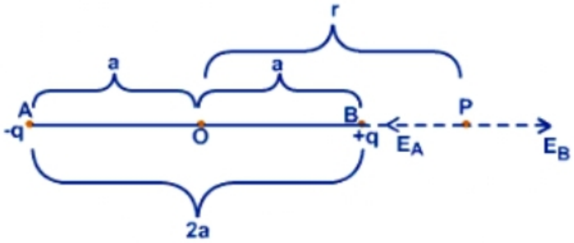
Suppose a test charge q_0 is placed at point p having r distance from centre of the dipole$\pm q\,.\overrightarrow{2a}$.

Now electric field at point p from charge –q at A is

$$|\vec{E}_A| = \frac{1}{4\pi\varepsilon_0}\frac{-q}{(r+a)^2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots1$$

& Electric field at P due to q charge at B is

$$|\vec{E}_B| = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots2$$



So the net electric field at P will be

$$|\vec{E}| = |\vec{E}_A| + |\vec{E}_B| = \frac{1}{4\pi\varepsilon_0}\frac{-q}{(r+a)^2} + \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{(r+a)^2-(r-a)^2}{[(r+a)\,(r-a)]^2}\right) = \frac{q}{4\pi\varepsilon_0}\left[\frac{r^2+a^2+2ar-r^2-a^2+2ar}{(r^2-a^2)^2}\right]$$

Or $$E = \frac{q.4ar}{4\pi\varepsilon_0(r^2-a^2)^2}$$

Or $$E = \frac{q\ 2a.\ 2r}{4\pi\varepsilon_0\ \ (r^2-a^2)^2} \qquad (\because\ 4ar = 2a.2r\ )$$

Or $$E = \frac{2pr}{4\pi\varepsilon_0(r^2-a^2)^2} \qquad (\because q.2a=P)$$

In vector form $$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{P}r}{(r^2-a^2)^2}$$

I,e electric field is in direction of electric dipole moment.

Special cases

- ✓ If dipole is small then $a << r$. Here a^2 can be neglected. In this case $\vec{E} = \frac{2\vec{P}r}{4\pi\varepsilon_0 r^4} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{P}}{r^3}$
- ✓ *Electric field varies as* $\frac{1}{r^3}$ *in case of electric dipole.*
- ✓ *If point p lies near the dipole then we can take.* $\vec{E} = \frac{2\vec{P}r}{4\pi\varepsilon_0 a^4}$

Question 19: Two charges 10 µC and −10 µC are placed at points A and B separated by a distance of 10 cm. Find the electric field at a point P on the perpendicular bisector of AB at a distance of 12 cm from its middle point.

$Ans : E = \frac{2pr}{4\pi\varepsilon_0(r^2-a^2)^2} = 8.2 \times 10^6 NC^{-1}$

23 Electric field at an equatorial point of a dipole:- M.Imp

Suppose a test charge q_0 is placed at point p equatorial to electric dipole ± q.2a.

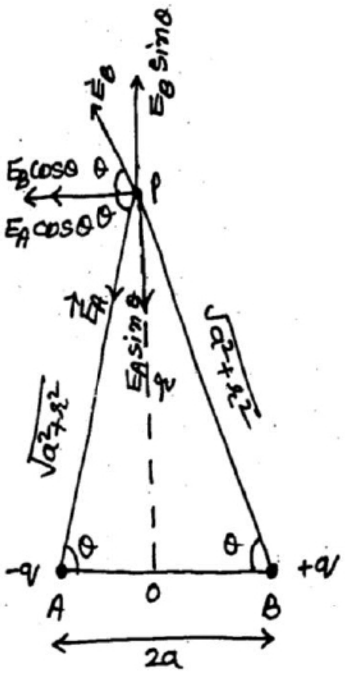
The distance of p from centre of the dipole o is r. As shown in fig. Now electric field E_A at p due to –q charge at A is toward $\overrightarrow{PA}$& electric field at p due to +q charge at B is toward $\overrightarrow{PB}$.

Such that $\left|\vec{E}_A\right| = \frac{1}{4\pi\varepsilon_0}\frac{q}{(PA)^2} \Rightarrow \left|\vec{E}_A\right| = \frac{1}{4\pi\varepsilon_0}\frac{q}{(\sqrt{a^2+r^2})^2}$……………….1

& $\left|\vec{E}_B\right| = \frac{1}{4\pi\varepsilon_0}\frac{q}{(PB)^2} \Rightarrow \left|\vec{E}_B\right| = \frac{1}{4\pi\varepsilon_0}\frac{q}{(\sqrt{a^2+r^2})^2}$………………..2

Cleary from equation (1) & (2) $\left|\vec{E}_A\right| = \left|\vec{E}_B\right|$

Now resoling $\left|\vec{E}_A\right|$ & $\left|\vec{E}_B\right|$ into rectangular components, we can see that $\left|\vec{E}_A\right|$ sinθ & $\left|\vec{E}_B\right|$ sinθ are equal and opposite so cancel out.



Net electric field is E = $\left|\vec{E}_A\right|$ cosθ + $\left|\vec{E}_B\right|$ cosθ

Or E =2$\left|\vec{E}_A\right|$ cosθ ………………..3 (∵$\left|\vec{E}_A\right| = \left|\vec{E}_B\right|$)

Here $\left|\vec{E}_A\right| = \frac{q}{4\pi\varepsilon_0(r^2+a^2)}$ & $\cos\theta = \frac{OA}{PA} = \frac{a}{\sqrt{a^2+r^2}}$

So from equation (3) E=2$\left|\vec{E}_A\right|$ cosθ = $2\frac{q}{4\pi\varepsilon_0(r^2+a^2)}\cdot\frac{a}{\sqrt{a^2+r^2}}$

$$= \frac{2qa}{4\pi\varepsilon_0(r^2+a^2)^{3/2}}$$

Or $$E = \frac{P}{4\pi\varepsilon_0(r^2+a^2)^{3/2}}$$

In vector from $$\vec{E} = \frac{-\vec{P}}{4\pi\varepsilon_0(r^2+a^2)^{3/2}}$$

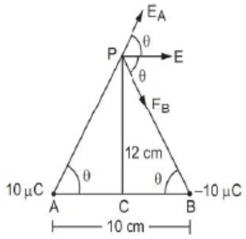
Here –ve sign indicate that electric field is in opposite direction to $\vec{P}$.

**If electric dipole is small then a<<r. Then electric field becomes* $\boldsymbol{\vec{E}} = \frac{-\vec{P}}{4\pi\varepsilon_0 r^3}$

Clearly $E \propto \frac{1}{r^3}$

Q 20: Two charges 10 μC and −10 μC are placed at points A and B separated by a distance of 10 cm. Find the electric field at a point P on the perpendicular bisector of AB at a distance of 12 cm from its middle .point.

Solution: $E = 4.1 \times 10^6 NC^{-1}$



*Relation between $\vec{E}$ axial &$\vec{E}$ equatorial for a short dipole

$$\frac{E_{\text{axial}}}{E_{\text{ equitonal}}} = \frac{\frac{2P}{4\pi\varepsilon_0 r^3}}{\frac{P}{4\pi\varepsilon_0 r^3}} = 2$$

Or $$\boldsymbol{E_{axial}} = \boldsymbol{2\ E_{equatorial}}$$

**In case of point plying nears the dipole we can take* $E_{equ}. = \frac{\vec{P}}{4\pi\varepsilon_0 a^3}$

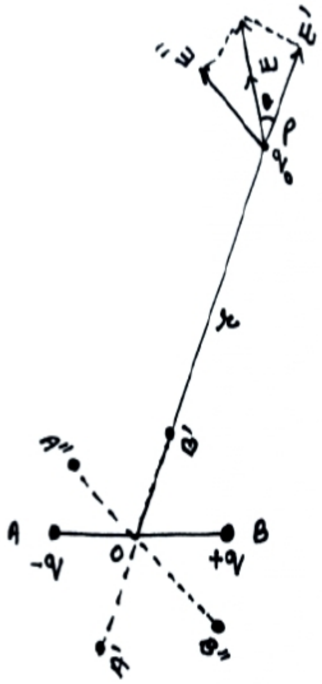
24.Electric Field at any Point of electric dipole:- imp

Suppose a test charge lying at any point P having distance r from the centre of the dipole.± q.2a inclined at an angle θ. Now to calculate electric field at point P due to dipole, resolve dipole into two components, such that point p lies at axial &equatorial points of the components.

Again suppose that point P lies at axial point of dipole A'B' &equatorial point of dipole A''B''.

Then Net electric field at point P is

$$\left|\vec{E}\right| = \sqrt{(E^{'} cos\theta)^2 + (E^{''}\ sin\theta)^2} \ldots\ldots\ldots\ldots\ldots..1$$



Here for sort dipole $E^{'}_{\boldsymbol{axial}} = \frac{2P}{4\pi\varepsilon_0\, r^3}$& $E''_{\boldsymbol{equatorial}} = \frac{P}{4\pi\varepsilon_0\, r^3}$

So $$\left|\vec{E}\right| = \sqrt{\left(\frac{2P\ \cos\theta}{4\pi\varepsilon_0 r^3}\right)^2 + \left(\frac{P\ sin\theta}{4\pi\varepsilon_0 r^3}\right)^2}$$

Or $$\left|\vec{E}\right| = \frac{P}{4\pi\varepsilon_0\, r^3}.(\sqrt{4cos^2\theta + sin^2\theta})$$

Or $$\left|\vec{E}\right| = \frac{P}{4\pi\varepsilon_0\, r^3}(\sqrt{3cos^2\theta + cos^2\theta + sin^2\theta})$$

Or $$\boxed{\left|\vec{E}\right| = \frac{P}{4\pi\varepsilon_0\, r^3}\sqrt{3cos^2\theta + 1}}$$

Special cases

- ✓ If *point P lies at axial point of electric dipole then* $\theta = 0^0 \Rightarrow \cos 0^0 = 1$

$$\Rightarrow \qquad E = \frac{2P}{4\pi\varepsilon_0 r^3}$$

- ✓ *If point P lies at equatorial point of electric dipole then* $\theta = 90^0 \Rightarrow \cos 90^0 = 0$

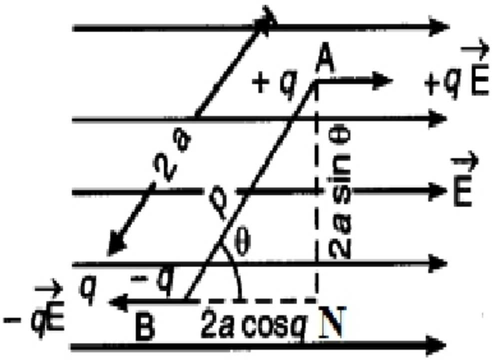
$$\Rightarrow \qquad E = \frac{P}{4\pi\varepsilon_0 r^3}$$

25. Torque on a electric dipole placed in uniform two dimensional electric field:- imp

Suppose a electric dipole ±q.2a is placed in a uniform two dimensional electric field E. Then dipole experience two equal & opposite forces in two directions which results into torque on the dipole.

$$Torque = force \times \perp distance$$

Or $\tau = \pm qE \times BN$



Where $BN = AB \sin\theta = 2a \sin\theta$

So $\tau = qE.\, 2a \sin\theta$

Or $\tau = PE\sin\theta$

In vector from $\vec{\tau} = \vec{P} \times \vec{E}$

The direction of $\vec{\tau}$ can be giving by ***right hand screw rule***.

- ✓ *Torque* has direction ⊥ to place containing $\vec{P}$ & $\vec{E}$.
- ✓ When dipole align to direction of electric field then $\theta = 0$ ⇒ $\sin 0^0 = 0$ So $\tau = o$
- ✓ When dipole lins⊥ to direction field then $\theta = 90°$ ⇒ $\sin 90^0 = 1$ ⇒ $\tau = PE$ (max)
- ✓ If $\theta = 90°$ & $E = 1\,N\,C^{-1}$ then $\tau = P$

Hence dipole moment may be defined as torque acting on an electric dipole placed ⊥ to electric field of unit strength.

❖ *Net force on electric dipole in uniform electric field is zero but $\tau \neq 0$*

Question 21: An electric dipole with dipole moment 4×10^{-9} C m is aligned at 30° with the direction of a uniform electric field of magnitude 5×10^4 N C^{-1}. Calculate the magnitude of the torque acting on the dipole?

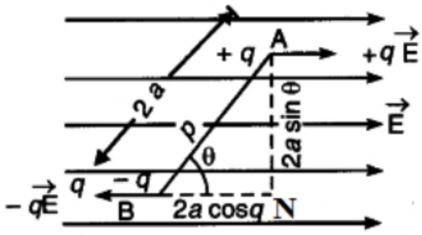
Ans: magnitude of the torque acting on the dipole is 10^{-4} N m.

26. Potential Energy of Electric Dipole in Uniform Two dimensional Electric field:- imp

Suppose an electric dipole is placed at an angleθ in a uniform two dimensional Electric field. Then torque experienced by electric dipole is

$$\tau = PE\sin\theta$$

Now suppose we want to rotate the dipole through small angle dθ. Then amount of work done to rotate the dipole is



$$dW = \tau.d\theta$$

Or $dW = PE\sin\theta\,.d\theta$

Now to calculate total work done to rotate dipole from θ_1 to θ_2 is

$$W = \int_{\theta_1}^{\theta_2} PE\sin\theta d\theta$$

$$= PE \int_{\theta_1}^{\theta_2} \sin\theta . d\theta$$

$$= PE[-\cos\theta]_{\theta_1}^{\theta_2}$$

$$= - PE [\cos\theta_2 - \cos\theta_1]$$

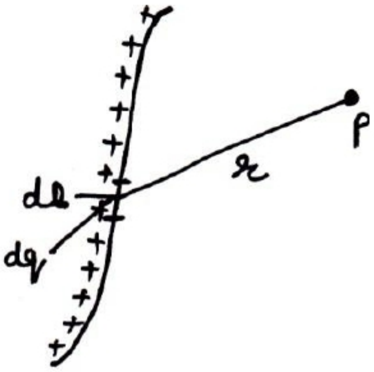
Or $$W = PE[\cos\theta_1 - \cos\theta_2]$$

This work is store in the form of potential energy so. $\mathbf{U = PE\ [\cos\theta_1 - \cos\theta_2]}$

27. Electric field at a point due to continuous distributions of charge:-

(i) electric field due to continuous line distribution of charge:

Suppose a point P having r distance from a point O on continuous line distribution of charge. Than small force at charge $\boldsymbol{q_0}$ placed at P due to small charge dq on small surface ds may be given by coulomb's law, $d\vec{F} = \frac{1}{4\pi\varepsilon_o}\frac{q_0 dq}{\vec{r}^2}\hat{r}$



But dq = λdl where λ is line charge density.

So $$d\vec{F} = \frac{1}{4\pi\varepsilon_o}\frac{q_0\lambda dl}{|\vec{r}|^2}\hat{r}$$

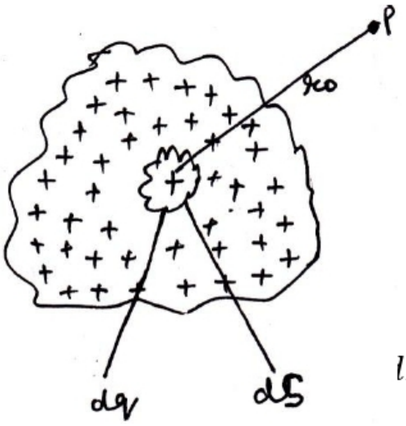
For complete length $$\vec{F} = \frac{1}{4\pi\varepsilon_o}\int\frac{q_0\lambda dl}{|\vec{r}|^2}\hat{r}$$

So electric Field at point P may be as given as $$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\int\frac{\lambda dl}{|\vec{r}|^2}\hat{\boldsymbol{r}}$$

Q.16 : An infinite line charge produces a field of 9 × 10^4 N/C at a distance of 2 cm. Calculate the linear charge density?

Ans: the linear charge density is 10 μC/m.

(ii) Electric field due to continuous surface distribution of charge:-



Suppose a test charge q_0 is placed at point p having r distance from small surface ds having charge dq. Then force on q_0 is

$$d\vec{F} = \frac{1}{4\pi\varepsilon_o}\frac{q_0 dq}{|\vec{r}|^2}\hat{r}$$

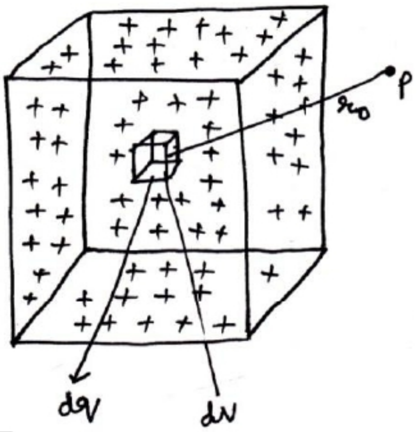
Where surface charge density $\sigma = \frac{dq}{ds} \Rightarrow dq = \sigma\ ds$

So the force at p due to complete surface is $\vec{F} = \frac{1}{4\pi\varepsilon_o}\int\frac{q_0\sigma\ ds}{|\vec{r}|^2}\hat{\boldsymbol{r}}$

Now electric field at point p is $$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\int\frac{\sigma\ ds}{|\vec{r}|^2}\hat{\boldsymbol{r}}$$

(iii) Electric field due to continuous volume distribution of charges.

Suppose a test charge q_0 is placed at point p having distance r from small volume dv having charge dq.



Then force on q_0 due to dq $\quad d\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_0 dq}{|\vec{r}|^2}\hat{r}$

Where volume charge density $\quad \rho = \frac{dq}{dv} \Rightarrow dq = \rho dv$

$$\Rightarrow d\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_0 \rho dv}{|\vec{r}|^2}\hat{r}$$

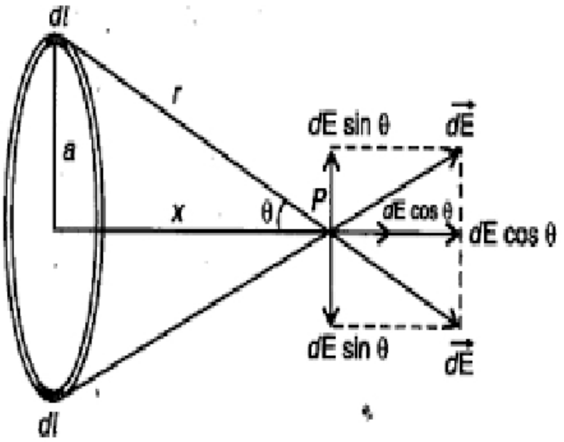
Hence force at p due to complete surface is $\quad \mathbf{F} = \frac{1}{4\pi\varepsilon_0}\int \frac{q_0 \rho dv}{|\vec{r}|^2}\hat{\boldsymbol{r}}$

So the electric field at point P will be $\quad \vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho dv}{|\vec{r}|^2}\hat{\boldsymbol{r}}$

28. Electric Field at a point on the axis of charged circular ring:-

Suppose a point P having x distance from centre of charged circular ring. Now small electric field dE at point P, due to small charge dq of length $d\ell$ 'is as shown in fig.

Here we can see that the components $dE \sin\theta$ are canceled out. The net electric field at P is



$$E = \int dE\cos\theta \ldots\ldots\ldots\ldots(1)$$

Here $\quad dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{(a^2+x^2)} \quad \& \quad \cos\theta = \frac{x}{\sqrt{a^2+x^2}}$

So from (1) $\quad E = \frac{1}{4\pi\varepsilon_0}\int \frac{dq\ .\ x}{(a^2+x^2)^{3/2}}$

$$= \frac{1}{4\pi\varepsilon_0}\frac{x}{(a^2+x^2)^{3/2}}\int dq$$

As q is uniformly distribution. The charge $dq = +\frac{q}{2\pi a}.\ dl$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{x}{(a^2+x^2)^{3/2}}.\frac{q}{2\pi a}\int dl$$

Or $\quad E = \frac{1}{4\pi\varepsilon_0}\frac{xq}{(a^2+x^2)^{3/2}} \qquad (\because \int dl = 2\pi a)$

- ✓ *If p lies for away then* $a << r$ *then* $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}$

This is same as in case of electric field due to point charge.