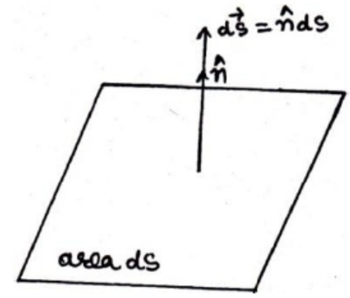
Chapter- 1 (c) Electrostatic Potential & Gauss's law

29. Area vector:-

In some cases of physics; we need to know not only the magnitude of a surface area but also its direction. The area vector is taken always perpendicular to the surface. As



$$d\vec{S} = \hat{n}\ ds$$

Here $\hat{n}$ is a unit vector ⊥ to the surface.

30. Electric Flux:- imp

The word flux comes from the Latin word 'fluere' means 'to flow'. Thus electric flux is a measure of flow of electric field through a surface. Or

Electric flux may be defined as the number of electric lines passing through a given area. It is a scalar quantity & denoted by $\emptyset_e$.

Suppose an area element $\overrightarrow{ds}$ is placed in a uniform electric field $\vec{E}$ making at angle θ.

Then $\emptyset_e = \vec{E}.\overrightarrow{ds}$

Or $\emptyset_e = Eds\ cos\theta$

Unit of $\emptyset_e$ $= NC^{-1} \times m^2 = Nm^2C^{-1}$

Special Cases:-

- *If area element is placed in plane of electric field then area vector become perpendicular to electric field. So*

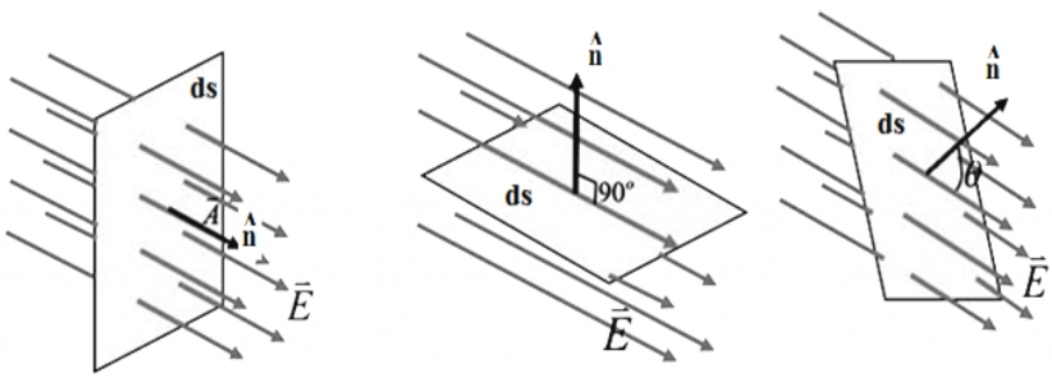
  $\theta = 90^0 \Rightarrow cos90^0 = 1$

  So $\Rightarrow \emptyset_e = Eds\ (0) = 0$

- *If area element is placed ⊥ to plane containing E. Then area vector become parallel to electric field.*

  $\theta = 0^0 \Rightarrow cos\ 0 = max = 1$

  $\emptyset_e = Eds\ (max)$



Q.21: Consider a uniform electric field E=3×10³îN/C. (a) What is the flux of this field through a square of 10cm on a side whose plane is parallel to the yz plane?

(b) What is the flux through the same square if the normal to its plane makes a 60° angle with the x; axis?

Ans (a) $\Phi = = 3 \times 10^3 \times 0.01 \times \cos 0° = 30\ \text{N m}^2/\text{C}$

(b) Plane makes an angle of 60° with the x; axis. Hence, θ=60° Flux, $\Phi = 3\times10^3 \times 0.01 \times \cos 60° = 15 \text{Nm}^2/\text{C}$

Question 22: What is the net flux of the uniform electric field of above question through a cube of side 20 cm oriented so that its faces are parallel to the coordinate planes?

Answer: All the faces of a cube are parallel to the coordinate axes. Therefore, the number of field lines entering the cube is equal to the number of field lines piercing out of the cube. As a result, net flux through the cube is zero.

Question23: **In a region of space the electric field is given by $\vec{E} = 8\hat{i} + 4\hat{j} + 3\hat{k}$. Calculate the electric flux Through a Surface of area 100 units in *x-y* plane.**

Solution: A surface of area 100 units in the *xy* plane is represented by an area vector $\vec{S} = 100\ \hat{k}$ (direction along the Normal to the area). The electric flux through the surface is given by $\phi_E = \vec{E}.\vec{S} = (8\hat{i} + 4\hat{j} + 3\hat{k}).(100\hat{k})$ **= 300 units**

Question24: **Calculate the electric flux through a cube of side '*a*' as shown, Where Ex = $bx^{1/2}$; $E_y = E_z = 0$, *a* = 10 cm and *b* = 800 N/C-m$^{1/2}$. (In Nm²/C)**

Solution: *x*-component is given by $E_x = bx^{1/2}$, where *b* = 800 N/C$^{1/2}$.

For the left face perpendicular to the *x*-axis, we have *x* = *a* = 10 cm, while for the right face *x* = 2*a* = 20cm.

Hence for the left face, the *x*-component of the field is $E_x = 800 \times (10 \times 10^{-2}\ \text{m})^{1/2} = 253$ N/c

For the right face, we have $E_x{}' = 800 \times (20 \times 10^{-2})^{1/2} = 358$ N/C

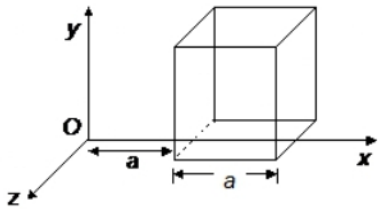
The area of each face is $S = 100\ \text{cm}^2 = 10^{-2} m^2$

Hence, the flux through the left face = $-E_xS = (253)\ (10^{-2}) = -2.53$ N-m²/C

The flux through the right face = $E_x{}'S = (358)\ (10^{-2}) = 3.58$ N-m²/C

The net flux through the other faces is zero, because $E_y = E_z = 0$

Hence, the net flux through the cube $\phi_E = 3.58 - 2.53 = \mathbf{1}$ (approx)



Q.25: A point charge causes an electric flux of –1.0 × 10³ Nm² /C to pass through a spherical Gaussian surface of 10.0 cm radius centered on the charge. (a) If the radius of the Gaussian surface were doubled, how much flux would pass through the surface? (b) What is the value of the point charge?

Answer: (a) Electric flux, $\Phi = -1.0 \times 10^3\ \text{N m}^2/\text{C}$ Radius of the Gaussian surface, r = 10.0 cm

Electric flux piercing out through a surface depends on the net charge enclosed inside a body. It does not depend on the size of the body. If the radius of the Gaussian surface is doubled, then the flux passing through the surface remains the same. i.e., $-1.0 \times 10^3\ \text{N m}^2/\text{C}$

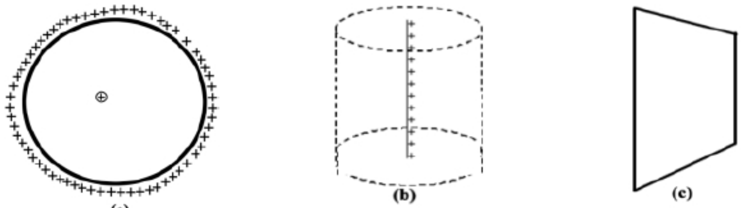
(b) Electric flux is given by the relation $\emptyset = \frac{q}{\varepsilon_0}$,

Where, q = Net charge enclosed by the spherical surface and ϵ_0 = Permittivity of free space = $8.854 \times 10^{-12}\ \text{N}^{-1}\text{C}^2\ \text{m}^{-2}$

$\therefore \quad q = -1.0 \times 10^3 \times 8.854 \times 10^{-12} = -8.854 \times 10^{-9}\ \text{C} = -8.854\ \text{nC}$

31. Gaussian Surface:

Any hypothetical closed surface around a charge having same electric field at all points is called Gaussian Surface. There are three types of Gaussian surface.



(a) (b) (c)

1. **Spherical Gaussian surface:-**

A Gaussian surface around a point charge is called spherical Gaussian surface. As shown in fig (a)

2. **Cylindrical Gaussian Surface:-**

A Gaussian surface around a line charge is called cylindrical Gaussian Surface. As shown in fig (b)

3. **Plane Gaussian surface:-**

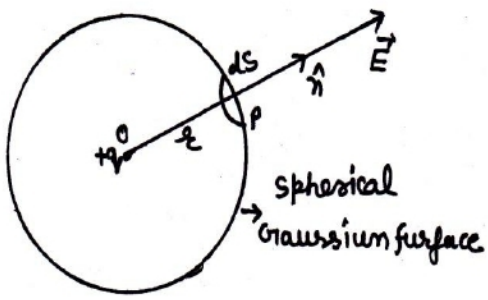
An infinite small part of spherical or cylindrical Gaussian surface is called plane Gaussians surface. As shown in fig (c)

32. Gauss Theorem:- M.Imp

According to Gauss theorem, electric flux or surface integral of electrical field over a closed surface is equal to $1/\varepsilon_0$ times the total charge enclosed by the surface.

I,e $$\phi_e = \oint_s \vec{E}.\vec{ds} = \frac{q}{\varepsilon_0}$$

Proof:- Suppose a point p having r distance from +q charge on the Gaussians Surface of small area ds. Then electric field at point p is $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$



& electric flux at point p is $\phi_e = \oint_s \vec{E}.\vec{ds}$

$$= \oint_s \vec{E}.\hat{n} ds\cos\theta$$

$$= \oint E\, ds \cos 0°$$

$$\phi_e = \oint E\, ds$$

$$= E \oint ds$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \times 4\pi r^2$$ (Here $\oint ds = 4\pi r^2$ is surface area of sphere)

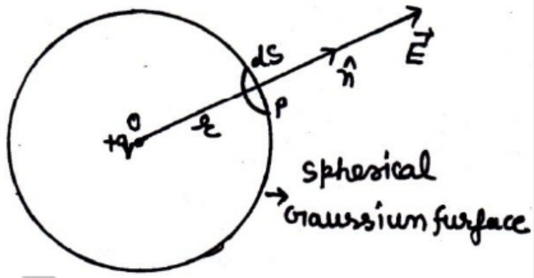
Or $$\boldsymbol{\phi_e = \oint \vec{E}.\vec{ds} = \frac{q}{\varepsilon_0}}$$

Hence Gauss theorem is proved

33. **Deduction of Coulomb's law from Gauss theorem:- Or**

Electric field at a point due to point charge:-

Suppose a point p on a Gaussian surface $\overrightarrow{ds}$ having charge $q_0$around a charge q. If r is the distance between q & q_0 then electric flux may be given by gauss theorem is



$$\phi_e = \oint \vec{E}.\overrightarrow{ds} = \frac{q}{\varepsilon_0}$$

Or $$\phi_e = \oint_s \vec{E}.\overrightarrow{ds} = \frac{q}{\varepsilon_0}$$

$$= \oint_s \vec{E}.\hat{n}ds\cos\theta = \frac{q}{\varepsilon_0}$$

$$= \oint Eds\cos 0° = \frac{q}{\varepsilon_0}$$

$$\phi_e = \oint Eds = \frac{q}{\varepsilon_0}$$

$$= E\oint ds = \frac{q}{\varepsilon_0}$$

Or $E\oint ds = \frac{q}{\varepsilon_0}$ Here $\oint ds$ = area of Gaussian sphere $= 4\pi r^2$

$$\Rightarrow \quad E \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

Or $$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

Now force may be given as $F = q_0E$

So $$F = \frac{1}{4\pi\varepsilon_0}\frac{q_0q}{r^2}$$

This is coulomb's law. Hence coulombs law can be obtained by Gauss theorem.

- ✓ *Gauss theorem is used to calculate electric field at a point, but Coulomb's cannot simplify problems related to electric field, Gauss's law is more easily, suitable & more useful in situations involving symmetry.*

34 APPLICATION OF GAUSS THEOREM M.Imp

(I). **Electric field due to an infinitely long charged wire:-**

Suppose a infinite long charged wire having length ℓ & charge q. Suppose a cylindrical Gaussian surface of radius r around the wire having three small area elements ds_1 , ds_2& ds_3 as shown in fig.

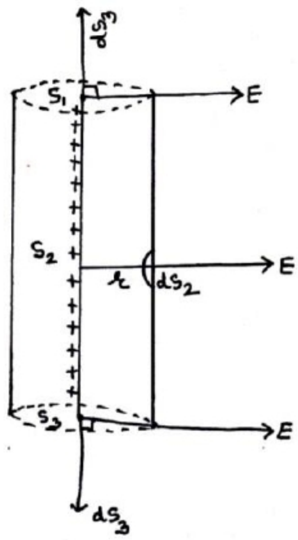
Now According to Gauss theorem

$$\phi_e = \oint_S \vec{E}.\overrightarrow{ds} = \oint_{S_1} \vec{E}.\overrightarrow{ds_1} + \oint_{S_2} \vec{E}.\overrightarrow{ds_3} + \oint_{S_3} \vec{E}.\overrightarrow{ds_3}$$

$$= \int_{s_1} Eds_1\cos 90° + \int_{s_2} Eds_2\cos 0° + \int_{s_3} Eds_3\cos 90° = \frac{q}{\varepsilon_0}$$

OR $\phi_e = \oint_S \vec{E}.\vec{ds} = \int_{s_2} Eds_2 = \frac{q}{\varepsilon_0}$

Or $\phi_e = E\int_{s_2} ds_2 = \frac{q}{\varepsilon_0}$



Here $\int ds_2$ = area of the cylinder = $2\pi r\ell$

So $\phi_e = E \times 2\pi r\ell = \frac{q}{\varepsilon_0}$

Or $E = \frac{q}{2\pi\varepsilon_0 rl}$

Or $E = \frac{\lambda}{2\pi\varepsilon_0 r}$ $(\because \lambda = (q/l))$

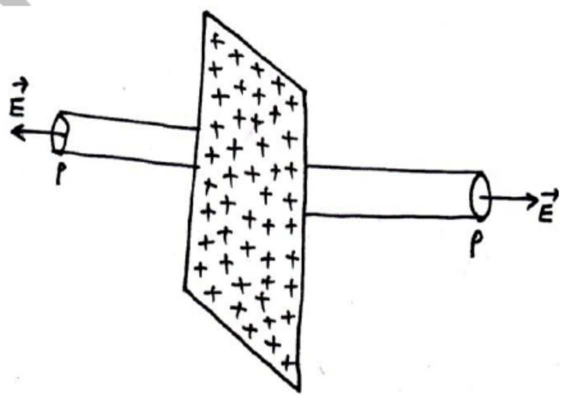
- *Thus electric field of a line charge is inversely proportional to the distance from the line charge.*

(II). **Electric Field due to uniformly charge infinite plane sheet:-**

Suppose a uniformly charged infinite plane sheet of charge having charge density σ. Again suppose that there are two Gaussian surfaces around the Plane sheet of charge having electric field E.

Now According to Gauss theorem

$$\phi_e = ES + ES = 2ES = \frac{q}{\varepsilon_0}$$



Or $2ES = \frac{q}{\varepsilon_0}$

Or $E = \frac{q}{2\varepsilon_0 s}$

Or $E = \frac{\sigma}{2\varepsilon_0}$ $(\because \frac{q}{s} = \sigma)$

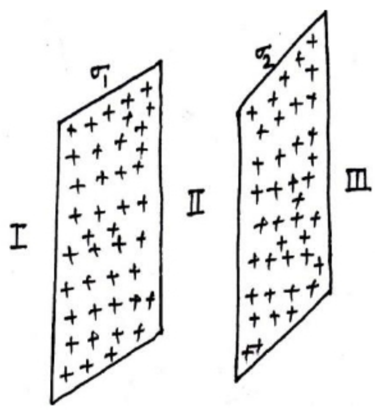
Clearly electric field due to plane sheet of charge does not depend on distance on from the plane sheet.

(III). **Electric Field due to two infinite plane sheet of charge :-**

Suppose there are two infinite plane sheets of charge having charge densities $\sigma_1$and σ_2. Again suppose that electric field of plane sheets of charge toward Right hand side is +ve & toward L.H.S. is –ve.

Now electric field in region (I) may be given as

$$E_I = \frac{-\sigma_1}{2\varepsilon_0} + \frac{-\sigma_2}{2\varepsilon_0} = -\frac{(\sigma_1+\sigma_2)}{2\varepsilon_0} \text{ ---------------------1}$$



Again electric field in region (II) may be given as

$$E_{II} = \frac{\sigma_1}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0} = \frac{1}{2\varepsilon_0}(\sigma_1 - \sigma_2) \text{------------2}$$

Similarly electric field in region (III) region me be given as

$$E_{III} = \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0} = \frac{1}{2\varepsilon_0}(\sigma_1 + \sigma_2) \text{-------}3$$

If $\sigma_1 = \sigma_2$ then eq^{n}1, 2 & 3 becomes.

$$E_I = \frac{-\sigma}{2\varepsilon_0} + \frac{-\sigma}{2\varepsilon_0} = \frac{-2\sigma}{2\varepsilon_0} = \frac{-\sigma}{\varepsilon_0}$$

&

$$E_{II} = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

$$E_{III} = \frac{2\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

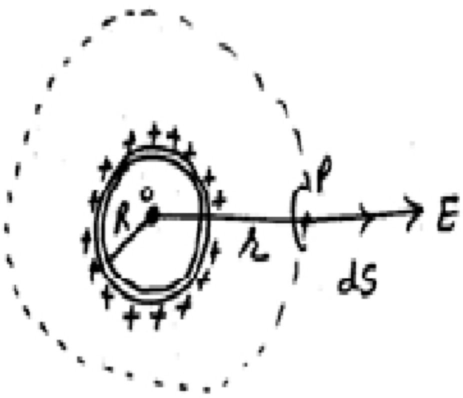
(IV). **<u>Electric Field due to a uniformly charged thin spherical shell.</u>**

Suppose a thin spherical charged shell having charge q & radius R.

A. When p point lies outside the spherical shell.

Suppose a point P on small Surface ds having charge q_0. Then electric field at point p is same as that of due to a point charge.

I,e $$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2} \quad (\text{For } r > R)$$



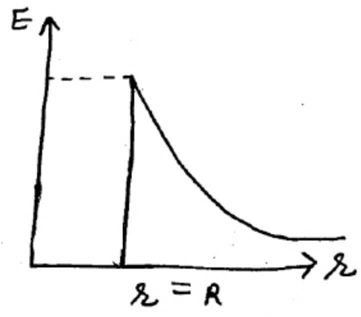
B. Now if point p lies on the spherical shell than according to Gauss theorem.

$$E \times 4\pi R^2 = \frac{q}{\varepsilon_0}$$

Or $$E = \frac{q}{4\pi\varepsilon_0 R^2} \quad (\text{For } r = R)$$

Or $$E = \frac{\sigma}{\varepsilon_0} \quad \left(\because \sigma = \frac{q}{4\pi R^2}\right)$$

C. When p point lies inside the spherical shell.



As we know that charge enclosed by a Gaussian surface is zero

So $$\phi_e = E\oint ds = o$$

Or $$E = o$$

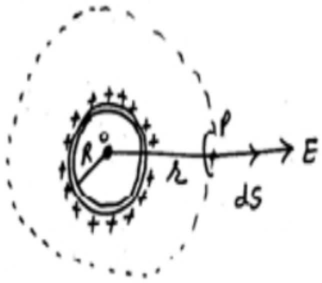
Hence *electric field inside a charged spherical shell is zero.*

Graphically variation of electric field with distance r is as shown fig electric field is zero when $r < R$, maximum at $r = R$ & decreases when $r < R$.

(V). **Electric Field due to a charged insulating sphere:-**

(A) When p point lies outside the sphere

Suppose a point P on small Surface ds having charge q_0. Then electric field at point p is same as that of due to a point charge



I,e $$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \quad \text{(For r >R) .}$$

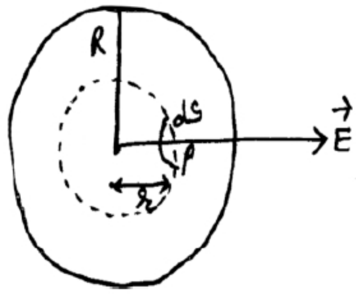
(B) Now if point p lies on the sphere than according to Gauss theorem.

So $$E \times 4\pi R^2 = \frac{q}{\varepsilon_0}$$

Or $$E = \frac{q}{4\pi\varepsilon_0 R^2} \quad (\text{for } r = R)$$

Or $$E = \frac{\sigma}{\varepsilon_0} \quad \left(\because \sigma = \frac{q}{4\pi R^2}\right)$$

(a) When p point lies inside the sphere



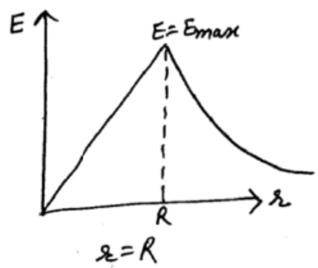
Suppose a charge sphere having charge q & radius R. Now electric field at a point p lying inside the sphere at point p having r distance from centre of the sphere is given by Gauss theorem.

$$\oint_s \vec{E}.\vec{ds} = \frac{q}{\varepsilon_0}$$

Cleary $$E = \oint_s ds = \frac{\frac{4}{3}\pi r^3\rho}{\varepsilon_0} \quad \left(\because \rho = \frac{q}{V} = \frac{q}{\frac{4\pi r^3}{3}}\right)$$

Or $$E \times 4\pi r^2 = \frac{\frac{4}{3}\pi r^3\rho}{\varepsilon_0}$$

$$E = \frac{\rho r}{3\varepsilon_0} \text{ ---------------------1}$$



Graphical variation of E with distance:

- ✓ If point p lies on the surface of the sphere then r = R. So from eq^n 1

$$\Rightarrow \quad E = \frac{\rho R}{3\varepsilon_0}(\text{maximum})$$

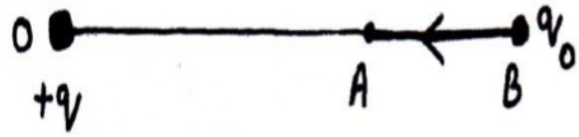
- ✓ If point p lies at centre of the sphere then r =0 so eq^n 1 becomes E= 0
- ✓ If point p lies outside the sphere then the case becomes same as in case of point charge.

As $E=\frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$ the variation of E with is as shown in graph.

Chapter 1 (d) Electrostatic Potential

35. **Electrostatic potential difference:-**

Suppose a point charge q_0 is placed at point B in the field of any other charge +q. Now amount of work done to move a charge from B to A is given by



$$\frac{W_{BA}}{q_0} = V_A - V_B = V_{BA}$$

Hence Electrostatic Potential difference may be defined as the amount of work done to move a unit +ve charge from one point to other point again the electrostatic forces without acceleration.

- It is assumed that test charge q_0 is so small that it does not disturb the source charge q.
- The external force is so small that it just balances the repulsive force between the charges & does not produce acceleration in source charge.

SI unit of potential difference is volt:- imp

i,e $$1 \text{ volt} = \frac{1 \text{ Jule}}{1\ Coulomb} = 1 \text{ NmC}^{-1} = 1 \text{ JC}^{-1}$$

Hence potential difference between two points is said to be one volt if one joule is the amount of work done to move one coulomb charge from one point to another point against the electrostatic forces without acceleration.

Electrostatic Potential:-

Electrostatic potential at a point is defined as the amount of work done to bring a charge from ∞ to that point against electrostatic force without acceleration.

At ∞ , $V_B = 0$

So $$\frac{W_{\infty A}}{q_0} = V_A$$

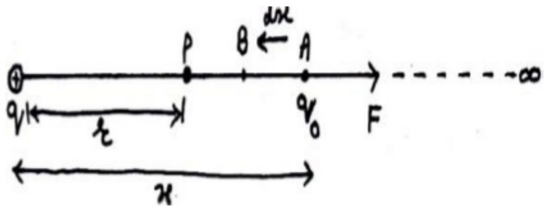
Unit of electrostatic potential is also volt.

$$1 \text{ volt} = \frac{1 \text{ Jule}}{1\ Coulomb} = 1 \text{ NmC}^{-1} = 1 \text{ JC}^{-1}$$

i,e *Hence electrostatic potential at a point is said to be one volt if one joule is the amount of work done to move one coulomb charge from ∞ to that point against electrostatic force without acceleration.*

36. **Electric Potential Due to a point charge:-** imp

Suppose a test charge q_0 is placed at point A & a charge + q is at point O. Now according to Coulomb's law the force between the charges is



$$F = \frac{1}{4\pi\varepsilon_0}\frac{q\,q_0}{x^2}$$

Now if we want to move charge from A to B having small distance dx between then. Then small amount of work dw may be given

As $$dw = \vec{F}.\overrightarrow{dx} = F\,dx\cos 180° = -F\,dx$$

So total work done to move charge from ∞ to p is

$$w = -\int_\infty^r F dx$$

$$= -\int_\infty^r \frac{1}{4\pi\varepsilon_0}\frac{q\,q_0}{x^2}\,dx$$

$$= \frac{-q\,q_0}{4\pi\varepsilon_0}\int_\infty^r \frac{1}{x^2}.\,dx$$

$$= \frac{-q\,q_0}{4\pi\varepsilon_0}\int_\infty^r x^{-2}.dx$$

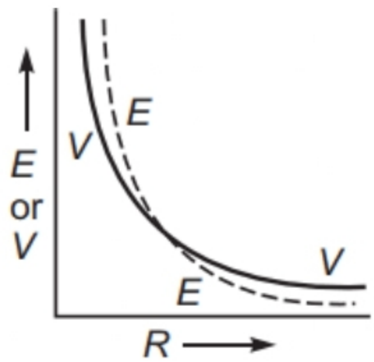
$$= \frac{-q\,q_0}{4\pi\varepsilon_0}\left[\frac{x^{-2+1}}{-2+1}\right]_\infty^r$$

$$= \frac{-q\,q_0}{4\pi\varepsilon_0}\left[\frac{x^{-1}}{-1}\right]_\infty^r$$

$$= \frac{-q\,q_0}{4\pi\varepsilon_0}\left[-\frac{1}{x}\right]_\infty^r$$

$$= \frac{q\,q_0}{4\pi\varepsilon_0}\left[\frac{1}{x}\right]_\infty^r$$

$$= \frac{+q\,q_0}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{\infty}\right]$$



Or $$W = \frac{q\,q_0}{4\pi\varepsilon_0 r}$$

As we know $\mathbf{V = \frac{W}{q_0}}$ so $\mathbf{V = \frac{W}{q_0} = \frac{q}{4\pi\varepsilon_0 r}}$

Cleary $V \propto \frac{1}{r}$ i.e. electric potential varies inversely with distance.

Question26: **The electric potential at point *A* is 200 V and at *B* is –400 V. Find the work done by an external force and Electrostatics force in moving charge of 2×10^{-8} C slowly from *B* to *A*. (in μJ)**

Solution: Here, $q_0 = 2 \times 10^{-8}$ C; $V_A = 200\,V$; $V_B = -400\,V$

Work done by the external force = $W_{B\to A} = q_0(V_A - V_B) = (2 \times 10^{-8})$ [(200 – (-400)]

Work done by the electric force = $-(W_{B\to A})_{\text{external}}$ = 12

Example 27: Two charges + 10 μC and + 20 μC are placed at a separation of 2 cm. Find the electric potential due to the pair at the middle point of the line joining the two charges.

Solution: Using the equation $V = \frac{Q}{4\pi\varepsilon_0 r}$

The potential due to + 10 μC is $V_1 = \frac{(10\times 10^{-6}C)\times(9\times10^{9}\ N\,m^2C^{-2})}{1\times10^{-2}\ m} = 9\ MV.$

The potential due to + 20 μC is $V2 = \frac{(20\times 10^{-6}C)\times(9\times10^{9}\ N\,m^2C^{-2})}{1\times10^{-2}\ m} = 18\ MV.$

The net potential at the given point is 9 MV + 18 MV = 27 MV.

If the charge distribution is continuous, we may use the technique of integration to find the electric potential.

Example 28: (a) Calculate the potential at a point P due to a charge of 4×10^{-7}C located 9 cm away.

(b) Hence obtain the work done in bringing a charge of 2×10^{-9} C from infinity to the point P. Does the answer depend on the path along which the charge is brought?

Solution (a) = 4×10^4 V (b) = 8×10^{-5} J

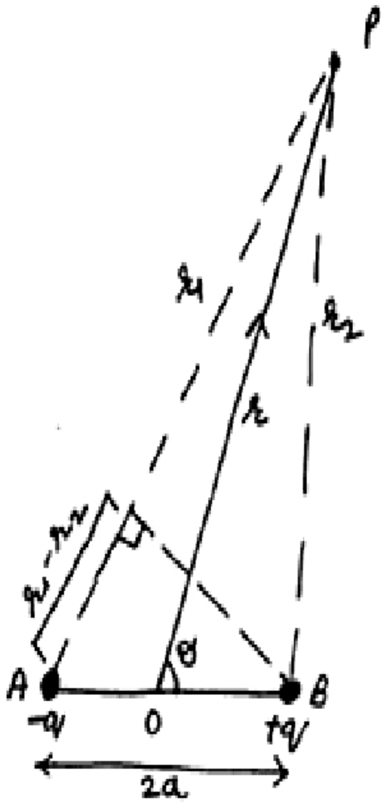
No, work done will be path independent. Any arbitrary infinitesimal path can be resolved into two perpendicular displacements: One along r and another perpendicular to r. The work done corresponding to the later will be zero.

Example 29; two charges 3×10^{-8} C and -2×10^{-8} C are located 15 cm apart. At what point on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

Ans electric potential is zero at 9 cm and 45 cm away from the positive charge on the side of the negative charge. Note that the formula for potential used in the calculation required choosing potential to be zero at infinity.

37. Electric Potential at any point due to a dipole:- imp

Suppose a test charge q_0 is placed at point P having r distance with the center of a dipole ± q. 2a placed at A & B. As shown in fig. Let AP = r_1 & BP = r_2.



Now net potential at p is

$$V = V_A + V_B$$

$$= \frac{-q}{4\pi\varepsilon_0 r_1} + \frac{q}{4\pi\varepsilon_0 r_2}$$

Or $$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{r_1-r_2}{r_1 r_2}\right] \text{----------------1}$$

If p point lies far away from the dipole than

$$r_1 - r_2 \simeq AB\cos\theta = 2a\cos\theta$$

& $$r_1 r_2 \simeq r^2$$

So eq^n 1 becomes $$V = \frac{q}{4\pi\varepsilon_0}.\frac{2a\cos\theta}{r^2} \text{--------------------2}$$

Or $$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{P}\cdot\hat{r}}{r^2}$$ (p= q.2a)

Or $$V = \frac{1}{4\pi\varepsilon_0}\frac{\vec{P}\cdot\vec{r}}{r^3} \text{----------------------3}$$

Special Cases:-

- If the point P lies on the axial of electric dipole than θ= 0° or 180° ⟹ cos0° = 1
  So from eq^n 2 $V = \frac{+p}{4\pi\varepsilon_0 r^2}$ i.e. Potential is maximum.
- When the p point lies on the equatorial line of electric dipole then Q= 90° ⇒ cos 90 ° = 0 ⇒ V= 0
  i.e. Potential at equatorial point of electric dipole is zero.

38. **Difference between electric potential of a dipole & A Single Charge.**

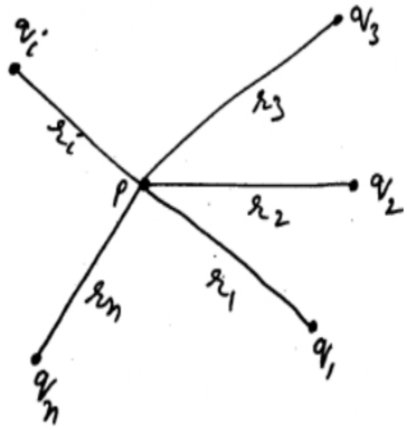
1. Potential due to dipole depends upon the distance & angle between dipole moment p & distance r where as potential due to single charge depend only on distance.
2. Potential due to dipole is cylindrical symmetric while potential due to point charge is spherical Symmetric.
3. Potential due to dipole varies as $\frac{1}{r^2}$ while potential due to single charge varies as $\frac{1}{r}$.

39. Electric Potential Due to a System of charges:-

Suppose q_1, q_2, q_3 ……..q_n charges having distances r_1, r_2, r_3 ……….. r_n from a point p. then total potential at p may be calculated as given below.

Potential at p due to q_1 charge is

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} \text{-------------------------}1$$



Again potential at p due to q_2 charge

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2} \text{-------------------------}2$$

Similarly potential at p due to q_3 charge

$$V_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3} \text{-------------------------}3$$

. .

. .

. .

$$V_n = \frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_n} \text{-------------------------}n$$

Adding all the eq^{n}s we get

$$V = V_1 + V_2 + V_3 \ldots\ldots\ldots V_n = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2} + \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3} \text{---------------} \frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_n}$$

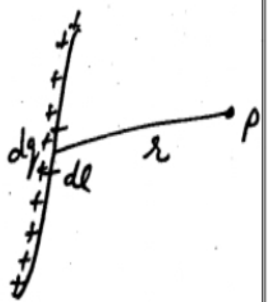
Or $$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} \ldots\ldots \frac{q_n}{r_n}\right)$$

Or $$\boldsymbol{V = \frac{1}{4\pi\varepsilon_0}\sum_{i=0}^{n}\frac{qi}{ri}}$$

40. **Electric Potential due to Continuous charge distributions:-**

(i) Suppose a point P having r distance from a continuous line distribution of charge.

Then potential at p due to small charge dq is



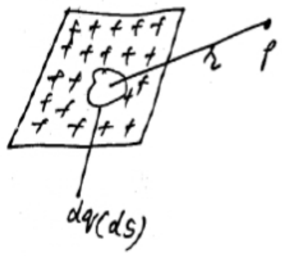
$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r}$$

Here $dq = \lambda\, dl$

So $$\boldsymbol{V = \frac{1}{4\pi\varepsilon_0}\int\frac{\lambda\, dl}{r}}$$

(ii) Suppose a point P having r distance from a continuous surface distribution of charge.

Then potential at p due to small charge dq is



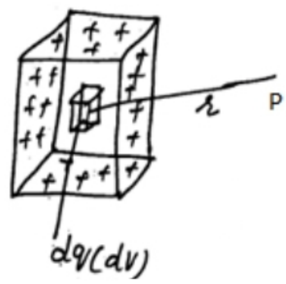
$$dv = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r}$$

Here $dq = \sigma\, ds$

So $$\mathbf{V = \frac{1}{4\pi\varepsilon_0}\int\frac{\sigma ds}{r}}$$

(iii) Suppose a point p having r distance from a continuous volume distribution of charge.

Now potential at p due to charge dq is



$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r}$$

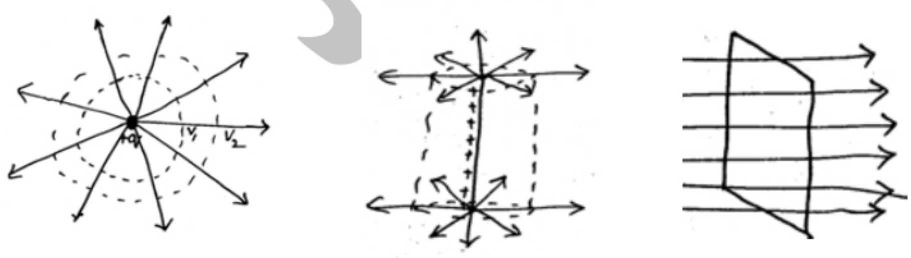
Here $dq = \rho\, dV$

So total potential at P becomes $$\mathbf{V = \frac{1}{4\pi\varepsilon_0}\int\frac{\rho dv}{r}}$$

41. **Equipotential Surface & their properties:-** M.Imp

Any surface which has same electric potential at every point on it is called equipotential surface.

There are three types of equipotential surfaces.



(i). **Spherical Equipotential surface:-**

An Equipotential surface around a point charge is of spherical in shape so called spherical equipotential surface.

(ii) **Cylindrical Equipotential Surface:-**

An Equipotential surface around a line charge is of cylindrical in shape so called cylindrical equipotential surface.

(iii) **Plane Equipotential Surface:-**

A infinite small part of spherical or cylindrical Equipotential surface becomes like a plane so called plane Equipotential surface.

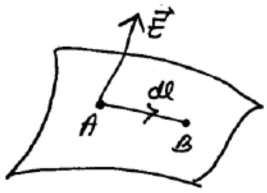
Properties:-

(i) **The work done to move a test charge on a Equipotential surface is zero:-**

Suppose a test charge is moved from A to B on an equipotential surface. Then amount of work done to move charge from A to B is

$$\frac{w_{AB}}{q_0} = V_B - V_A$$

But $V_A = V_B$ on an equipotential surface, because potential at every point in Equipotential surface is same



$$\Rightarrow \quad \frac{w_{AB}}{q_0} = 0$$

Here $q_0 \neq 0 \Rightarrow w_{AB} = 0$

Hence the work done to move a test charge on an equipotential surface is zero

(ii) No two Equipotential surfaces can intersect at each other:-

If two equipotential surfaces will intersect each other, then at that point there will be two values of electric potential which is not possible. Hence No two equipotential surface will intersect each other.

(iii) **Electric field is always normal to the equipotential surface:-**

As On Equipotential surface W= 0 $\Rightarrow W = \vec{F}.\overrightarrow{dr} = 0$

Or $q_0\vec{E}.\overrightarrow{dr} = 0$

Here $q_0 \neq 0 \Rightarrow \vec{E}.\overrightarrow{dr} = 0 \Rightarrow E\, dr\, cos\theta = 0$

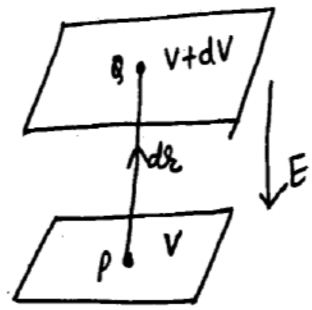
Or $\cos\theta = 0 \Rightarrow \theta = 90°$

Hence electric field is normal to the equipotential surface.

42. Relation between Electric Field and Potential Difference:- M.Imp

Suppose two equipotential surfaces having potential $V \& V + dV$. Now again suppose that a test charge q_0 is moved from point P to Q through small distance dr.

Then amount of work done to move charge is $dW = \vec{F}.\overrightarrow{dr}$



Or $dW = q_0\vec{E}.\overrightarrow{dr}$

Or $\frac{dW}{q_0} = \vec{E}.\overrightarrow{dr}$-------------------- (1)

Again we know that potential difference to move charge from P to Q is

$$\frac{dW}{q_0} = V + dV - V$$

Or $\frac{dW}{q_0} = dV$ --------------- (2)

Comparing eq^n(1) & (2) $\vec{E}.\overrightarrow{dr} = dV$

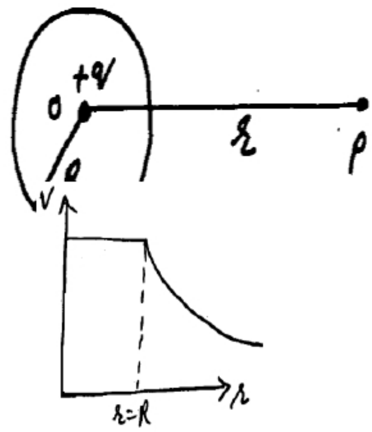
Or $Edr\cos 180^0 = dV$

$\Rightarrow$ $E = \frac{-dV}{dr}$ (∵here –ve sign show that E & dr are in opposite direction).

Hence electric field intensity is equal to negative gradient of potential difference.

43. Electric potential due to a uniformly charged spherical shell:-

(i) Suppose a point p having r distance from a shell of radius R & charge +q. Such that $r > R$ Then electric potential at P will be $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$



I.e. electrostatic potential is inversely proportional to distance

(ii) If point P lies on the surface of the shell then r=R

$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$ (electrostatic potential is maximum)

(iii) If point P lies inside the shell then electric field E=0.

As we know ⇒ $E = \frac{-dV}{dr} = 0$ ⇒ $V = constant$

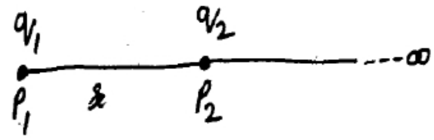
- So electric potential is constant, it remains same as that on the surface of the shell.
- *The relation between electric potential & distance of the charge is as shown in graph.*

44. **<u>Electric potential Energy:-</u>**

The electric potential energy of a system of charge may be defined as *the amount of work done in assembling charge at their location by bringing them from ∞ to a required point.*

Suppose a test charge is placed at point p_1 & a another charge q_2 is bring toward q_1 from ∞.Then electric potential between the charge is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}$$



& the amount of work done to bring charge q_2 from ∞ to P_2 is

$$W = Potential \times charge$$

So $$W = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}.q_2$$

This work is stored in the charge in from of potential energy. So potential energy of the system of charge is $\boldsymbol{U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}}$

⌗ Potential energy of a system of charge may be given as $U = \frac{1}{4\pi\varepsilon_0}\sum_{i,j}^{n}\frac{q_i q_j}{rij}$

Q.30: (a) Determine the electrostatic potential energy of a system consisting of two charges 7 μC and –2 μC (and with no external field) placed at (–9 cm, 0, 0) and (9 cm, 0, 0) respectively.

(b) How much work is required to separate the two charges infinitely away from each other?

(c) Suppose that the same system of charges is now placed in an external electric field $E = A\,(1/r^2)$;

$A = 9 \times 10^5\ C\,m^{-2}$. What would the electrostatic energy of the configuration be?

Solution (a) $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} = -0.7J$

(B) W = $U_2 - U_1$ = 0 – U = 0 – (–0.7) = 0.7 J

(c) = 70 -20 -0.7 =49.3 J

Chapter – 1 (E) – Capacitance

45. **Behavior of a conductor in electric field :-**

1. The net electric field inside a conductor is zero.
2. Just outside surface of a charged conductor, electric field is normal to the surface.
3. The net charged inside a conductor is zero & charge given to the conductor spread on its surface.
4. Potential is constant inside & on the surface of a conductor.
5. Electric field at the surface of a charged conductor is proportional to the surface charge density.
6. Electric field is zero in the cavity of a hollow charged conductor.

46. **Electrostatic shielding:-**

The phenomenon of making a hollow region of conductor having no any electric field inside it is called electrostatic shielding. It is based on the fact that electric field vanishes inside the cavity of a hollow conductor.

Uses of electrostatic shielding:-

1. In thunderstorm, during lighting it is safe to sit in car rather than near a tree or in open ground because metallic body of car act as electrostatic shielding from lightning.
2. Sensitive components of electronic devices are protected from external electric disturbance by placing them in metal shields.
3. In coaxial cables electrostatic shielding is used.

47. **Capacitance:-** imp

The ability of a body to store charge is called capacity or capacitance. If we add liquid in a container then the level of the liquid goes on rising similarly, if we give charge to a conductor, its potential keep on rising. Thus

$$Charge\ (Q) \propto Potential\ V$$

Or

$$Q = CV$$

Here C is constant of proportionality called capacity or capacitance of the conductor.

Or $$C = \frac{Q}{V}$$

The value of C depends upon:-

(i) The size & shape of the conductor.
(ii) The Nature of medium surrounding the conductor.
(iii) It does not depend upon material of the conductor by which it is formed and the value of charge and potential.

The SI unit of capacitance is farad (F)

$$\text{I,e} \qquad 1\, farad = \frac{1\, Coulomb}{1\, Volt}$$

Thus capacitance of a conductor is said to be one farad if one coulomb charge given to the capacitor raises its potential through one volt.

- The c, g, s unit of capacitance is one stat farad.

$$\text{i,e 1 stat farad} = \frac{1\, Stat\ Coulomb}{1\, stat\ Volt}$$

The capacitance of a conductor is said to be one stat farad if one stat coulomb charge raises the potential of 1 stat volt of conductor.

$$1\text{ Farad} = \frac{1\text{ C}}{1\text{ V}} = \frac{3\times10^{9}\text{stat Coulomb}}{(\frac{1}{300})\text{ stat Volt}} = 9\times10^{11}\text{ stat Farad}$$

Farad is a large unit of capacitance so smaller units are used is as

$$1\text{ micro farad} = 1\mu\text{ f} = 10^{-6}\text{F}$$

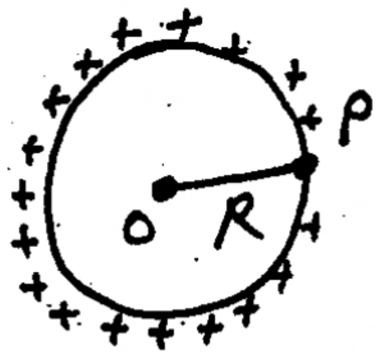
Or $\quad 1\text{ micro micro farad} = 1\text{ Pico farad} = 1\ \mu\mu\text{ f} = 1\text{ pF} = 10^{-12}\text{F}$

Dimensional formula: - $\quad C = \frac{Q}{V} = \frac{[AT]}{[ML^2T^{-3}A^{-1}]} = [\,M^{-1}L^{-2}T^{4}A^{2}\,]$

48. **<u>Capacitance of an isolated spherical capacitor :-</u>**

Suppose a isolated spherical capacitor of capacitance C. If q is the amount of charge given to the sphere, then potential at point p at the surface of the sphere is

$$V = \frac{q}{4\pi\varepsilon_0 R}$$



So capacitance $\quad C = \frac{q}{V} = \frac{q}{\frac{q}{4\pi\varepsilon_0 R}}$

$$= \frac{q\times4\pi\varepsilon_0 R}{q}$$

$$= 4\pi\varepsilon_0 R$$

So capacitance of isolated spherical capacitor is

$$C = 4\pi\varepsilon_0 R$$

- Here we can see that capacitance of the conductor depends only upon the radius of the conductor.

49. **Capacity of earth:-**

As we know radius of earth = 6.4×10^6 m

So its capacity C= $4\pi\varepsilon_0 R = \frac{6.4\times 10^6}{9\times 10^9} = 0.711 \times 10^{-3}\,F = 711 \times 10^{-6}\,F = 711\mu F$

- ✓ *Here it should be noted that capacity of earth is less than 1 F. So any body lying on the surface of earth does not have capacity of 1F.*

Q31: Calculate the radius of a conductor having capacitance 1F, and compare it with the radius of earth?

Ans. Here C = 1F so from relation C = $4\pi\varepsilon_0 R$

Radius of the planet may be calculated as $R_p = \frac{C}{4\pi\varepsilon_0} = \frac{1}{4\pi\varepsilon_0} = 9\times 10^9$m = 9×10^6 km

And we know that radius of earth is $R_e = 6.4\times 10^6$m

Comparing both we get $\frac{R_p}{R_e} = \frac{9\times 10^9}{6.4\times 10^6} \approx 1500$

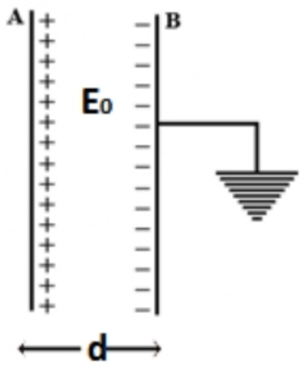
Thus a conductor having radius 1500 time more than radius of earth will have one farad capacitance.

50. Capacitor & its principle: imp

Capacitor: - *A capacitor is a device consists of two conductor separated by a small medium & is capable of store large amount of charge.*

Principle of capacitor:-

Suppose an uncharged plat B is placed near to a + vely charged plate A. Then due to ***induction of charges***, the face of plate B toward plate A acquire –ve charge & face away from the plate A acquire + ve charge. The charge on plate B increases when charge on plate A is increased & becomes constant at a certain limit.

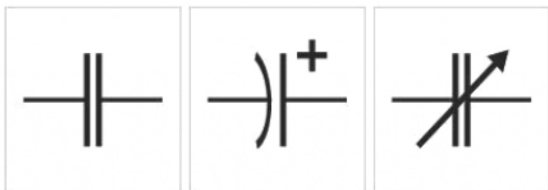


In this condition if we earth the +ve side of plate B, then +ve charge transfer into the earth. Due to this charge density of plate B decreases, now a time same more charge can be stored in plate B. Thus a large amount of charge can be stored in plate B. Thus a large amount of charge can be stored between arrangements of two conductors.

Symbol of capacitor:-

A capacitor of fix capacitance may be represented as

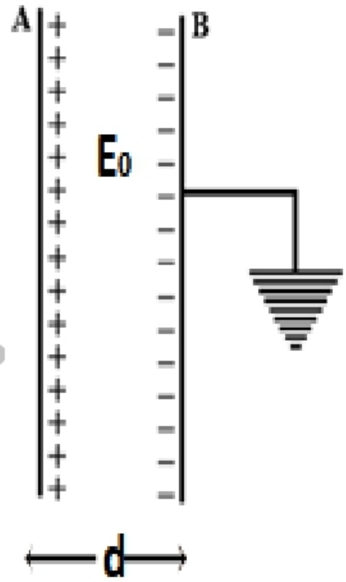
& a capacitor of variable capacitance may be represented as



Capacitor | Capacitor, polarized (American) | Capacitor, variable

51. **Capacitance of a parallel plate capacitor:-** imp

Suppose an arrangement of a parallel plate capacitor consist of two plates A & B separated by d distance. Again suppose that plate A is given +ve charge due to which –ve charge induced in plate B toward A & +ve charge away from A which is earthed.



Now potential between the plates may be given as

$$V = E_0 d$$

Or $$V = \frac{\sigma}{\varepsilon_0} d \qquad (\because E_0 = \frac{\sigma}{\varepsilon_0})$$

Here σ is uniform surface charge density.

So $$V = \frac{qd}{A\varepsilon_0} \qquad (\because \sigma = \frac{q}{A})$$

So capacitance of parallel plate capacitor becomes

$$C_0 = \frac{q}{V} = \frac{q}{\frac{qd}{A\varepsilon_0}} = \frac{qA\varepsilon_0}{q\, d}$$

Or $$\boldsymbol{C_0 = \frac{A\varepsilon_0}{d}}$$

Thus capacitance of a parallel plate capacitor depends upon.

1. Area of the plates, $(C_0 \propto A)$
2. Distance between the plates $(C_0) \propto \frac{1}{d})$
3. Permittivity of the mediums of the plates $(C_0 \propto \varepsilon_0)$

Question32: **The plates of a parallel plate capacitor are 5 mm apart and 2m² in area. The plates are in vacuum. A potential difference of 10,000 V is applied across a capacitor.**
Calculate:- (a) the capacitance: (in fm) (b) the charge on each plate ; (in nC)

Solution: (a) $C = \frac{\varepsilon_0 A}{d} = \frac{8.85\times10^{12}\times2}{5\times10^{3}} = 3540$

(b) $Q = CV = (0.00354 \times 10^{-6}) \times (10{,}000)$ = **3540**

The plate at higher potential has a positive charge of +3.54 μC and the plate at lower potential has a negative charge of – 3.54 μC.

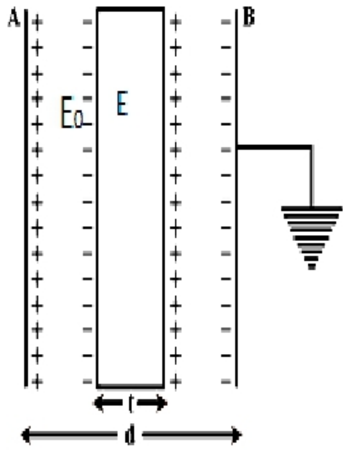
52 .**Parallel plate capacitor with dielectric slab:-** M.Imp

Suppose a parallel plate capacitor consist of two parallel Plates A & B separated by d distance apart.

Now again suppose that a dielectric slap of thickness t is inserted between the plates. If E_0 is electric field between the Plates & E is electric field in dielectric slap then potential between the plates is given by $$V = E_0\,(d - t) + Et \;\ldots\ldots\ldots\ldots\ldots\ldots 1$$

Also we know that $\frac{E_o}{E} = K$ = dielectric constant

$$\Rightarrow \qquad E = \frac{E_o}{K}$$



Using in eq^n 1 we get $V = E_0\left(d - t\right) + \frac{E_o}{K}t$

$$= E_0\left(d - t + \frac{t}{K}\right)$$

$$V = E_0\left[d - t\left(1 - \frac{1}{K}\right)\right]$$

$$= \frac{\sigma}{\varepsilon_0}\left[d - t\left(1 - \frac{1}{K}\right)\right] \qquad \left(\because E_0 = \frac{\sigma}{\varepsilon_0}\right)$$

Or $$= \frac{q}{A\varepsilon_0}\left[d - t\left(1 - \frac{1}{K}\right)\right] \qquad \left(\because \sigma = \frac{q}{A}\right)$$

Or $$V = \frac{qd}{A\varepsilon_0}\left[1 - \frac{t}{d}\left(1 - \frac{1}{K}\right)\right]$$

So capacitance becomes $$C = \frac{q}{v} = \frac{q}{\frac{qd}{A\varepsilon_0}\left[1 - \frac{t}{d}\left(1 - \frac{1}{K}\right)\right]} = \frac{q \quad .A\varepsilon_0}{qd\left[1 - \frac{t}{d}\left(1 - \frac{1}{k}\right)\right]}$$

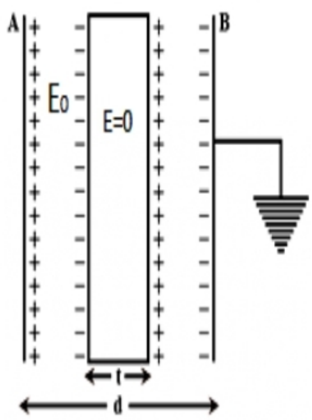
Or $$C = \frac{A\varepsilon_0}{d\left[1 - \frac{t}{d}\left(1 - \frac{1}{k}\right)\right]}$$

Or $$\mathbf{C} = \frac{C_0}{1 - \frac{t}{d}\left(1 - \frac{1}{k}\right)} \qquad \left(\because C_o = \frac{A\varepsilon_0}{d}\right)$$

- *Clearly $C > C_o$ hence capacitance of capacitor increases when a dielectric slab is placed between the capacitors.*

53. **Parallel Plate Capacitor With Conduction Slab:-** imp

Suppose a parallel plate capacitor consists of two parallel plates A & B separated by d distance apart. Again suppose that a conduction slab of thickness t is placed between the plates. As electric field between the plates of capacitor is E_o & in conduction slab is zero, So potential difference between the plates may be given as.



$$V = E_o(d - t)$$

Or $$V = \frac{\sigma}{\varepsilon_0}(d - t) \qquad \left(\because E_o = \frac{\sigma}{\varepsilon_0}\right)$$

Or $$V = \frac{qd}{A\varepsilon_0}\left(1 - \frac{t}{d}\right)$$

So capacitance of parallel plate capacitor becomes

$$C = \frac{q}{V} = \frac{q}{\frac{qd}{A\varepsilon_0}\left(1 - \frac{t}{d}\right)}$$

Or $$= \frac{q \quad A\varepsilon_0}{qd\left(1 - \frac{t}{d}\right)}$$

Or $$C = \frac{A\varepsilon_0}{d\ (1-\frac{t}{d})}$$

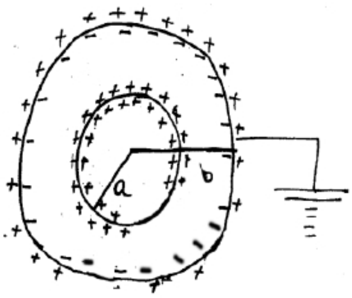
Or $$\boldsymbol{C = \frac{C_0}{1-\frac{t}{d}}} \qquad (\because C_o = \frac{A\varepsilon_0}{d} = \text{capacitance of parallel plate capacitor})$$

- Clearly $C > C_o$ *Hence capacitance of parallel plate capacitor increases when a conducting slab is placed between the plates.*

00. **Capacitance of a spherical capacitor** **(not directly in syllabus)**

Suppose a spherical capacitor consist of two concentric rings of radius a & b such that a<b. Now if +q charge is given to inner sphere then –q charge induces on outer sphere then potential difference between the sphere is

$$V = \frac{q}{4\pi\varepsilon_0 r_a} - \frac{q}{4\pi\varepsilon_0 r_b} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$



So capacitance of capacitors becomes $C = \frac{q}{V} = \frac{q.\ \ 4\pi\varepsilon_0}{q\left(\frac{1}{r_a} - \frac{q}{r_b}\right)}$

Or $$C = \frac{4\pi\varepsilon_0}{\left(\frac{r_b - r_a}{r_a r_b}\right)} = \frac{4\pi\varepsilon_0 r_a r_b}{(r_b - r_a)}$$

0. **Cylindrical Capacitor:-** **(not directly in syllabus)**

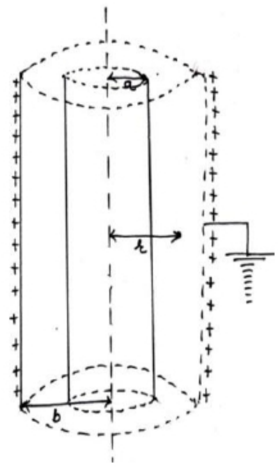
Suppose a cylindrical capacitor consist of two Cylinders of radius a & b. Now electric Field at any point p having r distance from the axis of cylinder is

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Now potential difference between two cylinder is

$$V = -\int_a^b \vec{E}.\ \vec{dr}$$

$$= -\int_a^b Edr\cos 180 = \int_a^b Edr$$



Or $$V = \int_a^b \frac{\lambda}{2\pi\varepsilon_0 r}.dr$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\int_a^b \frac{1}{r}.dr$$

Or $$V = \frac{\lambda}{2\pi\varepsilon_0}[\log r]_a^b$$

$$= \frac{\lambda}{2\pi\varepsilon_0}[\log b - \log a]$$

$$V = \frac{q}{2\pi\varepsilon_0 L}\log\frac{b}{a} \qquad (\because \lambda = \frac{q}{L})$$

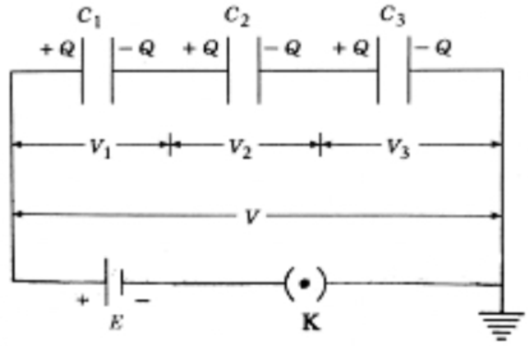
So capacitance $$\boldsymbol{C = \frac{q}{V} = \frac{q.\ \ 2\pi\varepsilon_0 L}{q\ \ \log\frac{b}{a}} = \frac{2\pi\varepsilon_0 L}{\log\frac{b}{a}}}$$

54. **Combinations of Capacitors:-** M.Imp

(i) **Capacitors in series:-**

When the negative plate of one capacitor is connected to the positive plate of the Second & negative of the second to the positive of third & so on, then the Capacitors are said to be connected in series.

Suppose three capacitors C_1, C_2, & C_3 are connected in series & q is the amount of charge stored in each capacitor. Then total rise in potential is



Capacitors in series

$$V = V_1 + V_2 + V_3.$$

Where $V_1 = \frac{q}{C_1}, V_2 = \frac{q}{C_2}, V_3 = \frac{q}{C_3}$

$\Rightarrow$ $\frac{q}{C_s} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$

$\Rightarrow$ $\frac{q}{C_s} = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$

Or $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

Here C_s is the equivalent capacitance due to series combination.

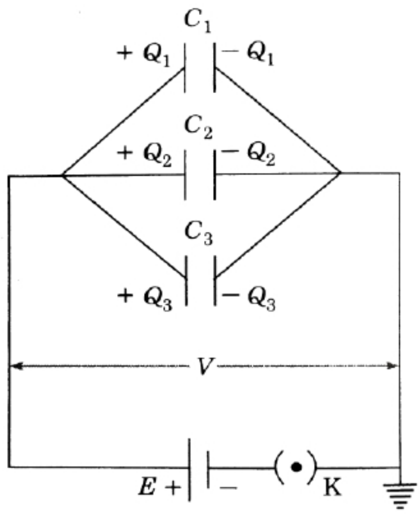
Hence equivalent capacitance in series combination is equal to the sum of reciprocal of the all the capacitor connected in series.

✓ Equivalent capacitance is smaller than the smallest capacitor.

(ii) **Capacitor's in parallel:-**

When +ve plates of all the capacitors are connected to one common point &- ve plates of all the capacitors are connected to other common point then the combinations of capacitor is called parallel combination.

Suppose C_1, C_2, C_3 are three capacitor Connected in parallel. If V is the potential across each capacitor, then total charge is



$$q = q_1 + q_2 + q_3$$

But $q_1 = C_1V$ $q_2 = C_2V$, $q_3 = C_3V$

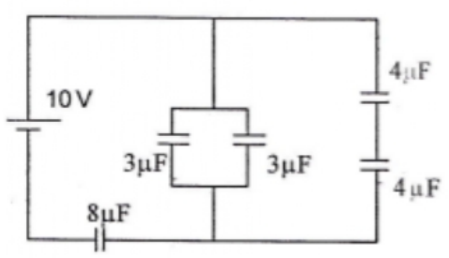
Then $C_PV = C_1V + C_2V + C_3V$

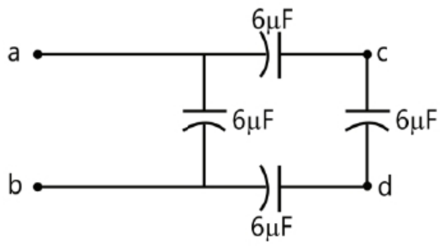
Or $C_P = C_1 + C_2 + C_3$

Here C_P is the equivalent capacitance due to parallel combination.

- Hence equivalent capacitance is equal to the sum of individual capacitance.
- Equivalent capacitance is larger than the largest individual capacitance.

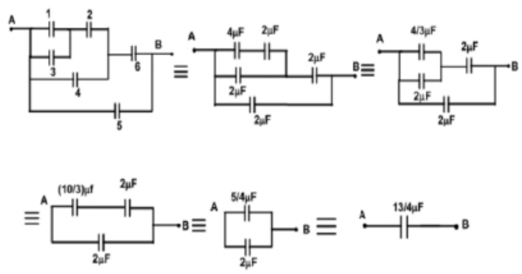
Q 33: Find the equivalent capacitance in circuit and total charge



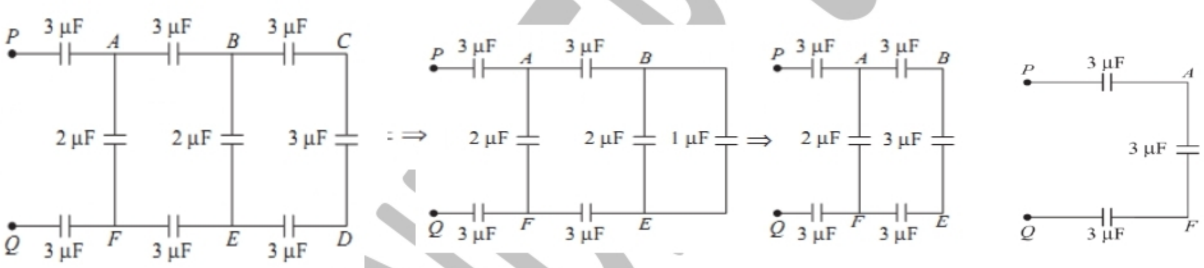


Q 34: Find the equivalent capacitance in circuit if each capacitor is of 2μf

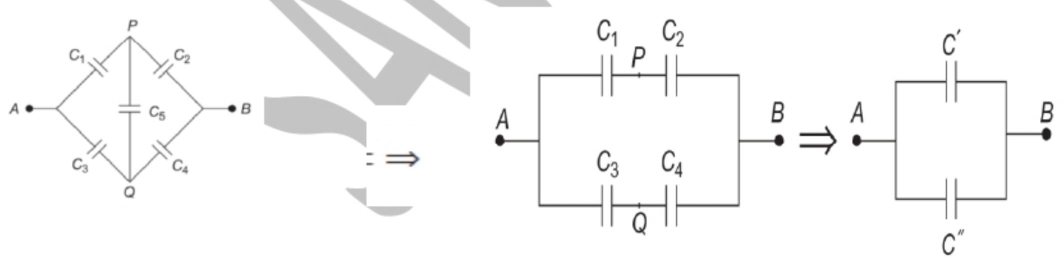
Ans



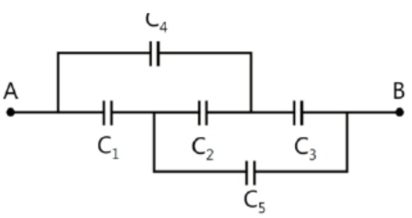
Q 35 :Find the equivalent capacitance in circuit

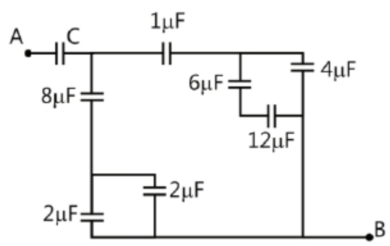


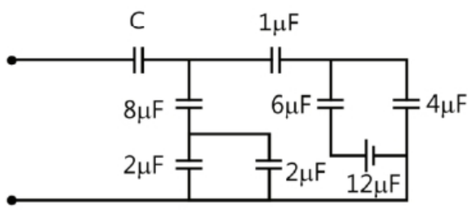
Q 36: Find the equivalent capacitance in circuit if each capacitor is of 2μf



Q 37: Find the equivalent capacitance in circuit if each capacitor is of 2μf

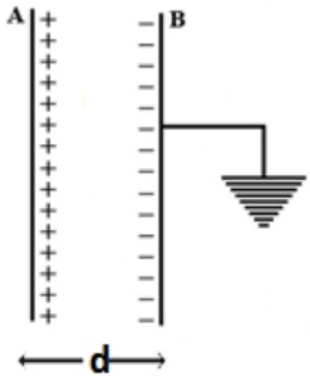






55. **Energy Stored in a capacitor:-** M.Imp

Suppose a parallel plate capacitor consist of two Parallel plates A & B. Initially both plates are neutral. Now suppose +q charge is transferred from plate B to plate A. Now to transfer some more charge dq from B to A, the work has to be done.



As $$dW = V.dq$$

$$\Rightarrow dW = \frac{q}{c}.dq$$

Now total amount of work done to move charge +Q from B to A is

$$W = \int_0^Q \frac{q}{c}\,.dq$$

Or $$W = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q = \frac{1}{C}\left[\frac{Q^2}{2}\right]$$

Or $$W = \left[\frac{Q^2}{2C}\right]$$

This work will store in the capacitor in from of energy so,

$$U = \frac{Q^2}{2C} \text{ ---------------------------- } 1$$

As $Q = CV \Rightarrow U = \frac{(CV)^2}{2C} = \frac{1}{2}CV^2$ ---------------2

Also $C = \frac{q}{V} \Rightarrow U = \frac{Q^2}{2\frac{Q}{V}} = \frac{1}{2}QV$ --------------------3

So we may write $$\mathbf{U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}qV}$$

Question38: **A parallel plate capacitor has plates of area 4 m^2 separated by a distance of 0.5 mm. The capacitor is Connected across a cell of emf 100 V. Find the energy store in the capacitor (in mJ) if a dielectric slab of Dielectric Strength 3 thicknesses 0.5 mm is inserted inside this capacitor after it has been disconnected from the Cell.**

Solution: $C = \frac{K\varepsilon_0 A}{d} = KC_0 = \mathbf{0.2124\ \mu F}$

$V = \frac{Q}{C} = \frac{Q_0}{KC_0} = \frac{V_0}{K} = \frac{\mathbf{100}}{\mathbf{3}}V$

$U = \frac{Q_0^2}{2C} = \frac{Q_0^2}{2KC_0} = \frac{U_0}{K} = \mathbf{118}$

56 . **Common potential:-**

Suppose two capacitor of capacitance C_1 & C_2 are connected in parallel, so total charge on the combination of capacitor is $Q = C_1 V_1 + C_2 V_2$

So the common potential of the combination of the charge is $V = \frac{\text{total charge}}{\text{totat capacitance}} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$

57. **Lose of energy in shearing of capacitor:-**

Suppose two capacitance C_1& C_2 having potential V_1& V_2. Now if capacitors are not joined together then the total energy of the capacitors may be given as

$$U = \frac{1}{2} C_1V_1^2 + \frac{1}{2} C_2 V_2^2 \text{ ----------------------------------1}$$

Now if capacitor are connected together than the energy of capacitor may be given by

$$U_1 = \frac{1}{2} C_1V_2 + \frac{1}{2} C_2 V_2 = \frac{1}{2} V_2 (C_1 + C_2) \text{ ----------------------2}$$

Where V is common potential of the combination

The loss of energy in shearing of the capacitor can be given by

$$\Delta U = U - U' = (\frac{1}{2} C_1V_1^2 + \frac{1}{2} C_2 V_2^{2)} - \frac{1}{2} V^2 (C_1 + C_2)$$

$$= (\frac{1}{2} C_1V_1^2 + \frac{1}{2} C_2 V_2^2) - \frac{1}{2} V^2 (C_1 + C_2)$$

$$= \frac{1}{2}\{ (C_1V_1^2 + C_2 V_2^2) - [\frac{C_1V_1 + C_2V_2}{C_1+C_2}]^2 (C_1 + C_2)\}$$

$$= \frac{1}{2}\{ (C_1V_1^2 + C_2 V_2^2) - \frac{(C_1V_1+C_2V_2)^2}{C_1+C_2}\}$$

$$= \frac{1}{2}\{\frac{(C_1+C_2)\,(C_1V_1^2 + C_2V_2^2) - (C_1V_1+C_2V_2)^2}{C_1+C_2}\}$$

$$= \frac{1}{2}\{\frac{(C_1^2V_1^2 + C_1C_2V_2^2 + C_1C_2V_1^2 + C_2^2V_2^2) - (C_1^2V_1^2 + C_2^2V_2^2 + 2C_1C_2V_1V_2)}{C_1+C_2}\}$$

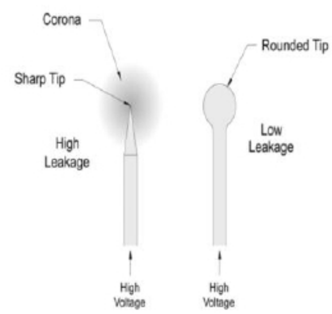
$$= \frac{1}{2}\{\frac{C_1^2V_1^2 + C_1C_2V_2^2 + C_1C_2V_1^2 + C_2^2V_2^2 - C_1^2V_1^2 - C_2^2V_2^2 - 2C_1C_2V_1V_2)}{C_1+C_2}\}$$

$$= \frac{1}{2}\{\frac{C_1C_2V_2^2 + C_1C_2V_1^2 - 2C_1C_2V_1V_2)}{C_1+C_2}\} = \frac{1}{2}\{\frac{C_1C_2(V_1^2+V_2^2-2V_1V_2)}{C_1+C_2}\}$$

$$\mathbf{\Delta U = \frac{1}{2}\{\frac{C_1C_2(V_1-V_2)^2}{C_1+C_2}\}}$$

- ✓ Here we can see that above value is positive. Hence we can say that there is a loose of energy in shearing of capacitors.

58. **Action of sharp point or corona discharge:-** imp



According to corona discharge or action of sharp point, if charge given to a sharp point of negligible area then density of charge becomes very high near the sharp point due to which the gasses in the neighboring of sharp point becomes ionized. As $\sigma = \frac{q}{A}$

- ➢ *Small the area larger is the charge density*

59. **Lightning conductor:-**

A lightning conductor is used to protect buildings from lightning. A lightning conductor is consisting to large number of sharp points. When a cloud of negatively charge passes near to it, than due to induction of charge + ve charge induces on it, which immediately passes into the earth. Thus buildings remain safe.

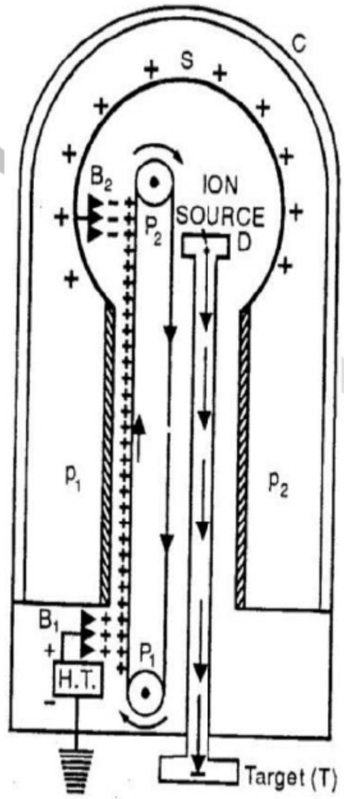
60. **Ven de Graff Generator:-** imp

It is a device which is used to make high potential difference of order of 10^7 volts, which is used to accelerate charge particles like electron, proton, ions etc.

Principle:-

(i) It is based on the action of sharp point or corona discharge.

(ii) That charge given to a hollow conductor transfers to its outer surface & distributes uniformly over it.



Construction:-

The Ven-de-Graff Generator is consisting of a large spherical conducting shell. A long narrow belt of insulating materials, like rubber or silk wounded around a pulley P_1 & P_2, near the bottom & top of pulley two sharp comb B_1& B_2 are fixed. The ***spray comb*** B_1 is connected to the High Tension battery (10kV) & ***collecting comb*** B_2 is connected to spherical shell. A shielding is done around the device to protect from radiations. A discharge tube is also fitted in the device to accelerate the charge particle, which is used to hit the target.

Working:-

When +ve charge is sprayed from comb B_1 to insulating belt then collecting comb B_2 becomes –vely charge due to induction of charge & sphere becomes + vely charged. In each rotation of belt sphere becomes more &more + vely charge. Particle inside the discharge tube acquire sufficient acceleration due to repulsion of large charge density on the sphere & may hit a target.

61. **Electric susceptibility:-**

The polarization $\vec{P}$ is directly proportional to the applied electric field $\vec{E}$ across a dielectric slab.

i, e $\quad \vec{P} \propto \vec{E} \quad$ Or $\quad \vec{P} = \varepsilon_0 \chi \vec{E}$

Here χ is called electric susceptibility. It has no dimensions $\quad \chi = \frac{\vec{P}}{\varepsilon_0 \vec{E}}$

Ration between K &χ

The net electric field in a polarized dielectric is $\vec{E} = \vec{E}_0 - \vec{E}_p$

But $\vec{E}_P = \frac{\sigma \vec{P}}{\varepsilon_0} = \frac{\vec{P}}{\varepsilon_0}$

$$\vec{E} = \vec{E}_0 - \frac{\vec{P}}{\varepsilon_0}$$

Or $\vec{E} = \vec{E}_0 - \frac{\varepsilon_0 \chi \vec{E}}{\varepsilon_0}$

Dividing both side by $\vec{E}$ we get $1 = \frac{\vec{E}_0}{\vec{E}} - \chi$ Or $1 = K - \chi$ Or $K = 1 + \chi$

62. **Dielectric Strength:-**

The maximum electric field that can exist in a dielectric without causing the breakdown of its insulating property is called dielectric strength of the material.

SOME IMPORTANT MCQ

1. A coulomb is the same as:

A. an ampere/second B. half an ampere·second^2 C. an ampere/meter^2 **D. an ampere·second**

2. A kiloampere·hour is a unit of:

A. current B. charge per time C. Power **D. charge**

3. The total negative charge on the electrons in 1mol of helium (atomic number 2, molar mass4) is:

A. $4.8 \times 10^4\ C$ B. $9.6 \times 10^4\ C$ **C. $1.9 \times 10^5\ C$** D. $3.8 \times 10^5\ C$

4. A small object has charge Q. Charge q is removed from it and placed on a second small object.The two objects are placed 1m apart. For the force that each object exerts on the other to be a maximum. q Should be:

A. 2Q B. Q **C. Q/2** D. Q/4

5. Two small charged objects attract each other with a force F when separated by a distance d. If the charge on each object is reduced to one-fourth of its original value and the distance between them is reduced to d/2 the force becomes:

A. F/16 B. F/8 **C. F/4** D. F/2

6. As used in the definition of electric field, a "test charge":

A. has zero charge
B. has charge of magnitude 1C
C. has charge of magnitude $1.6 \times 10^{-19}\ C$
D. none of the above

7. The electric field at a distance of 10 cm from an isolated point particle with a charge of 2×10^{-9} C is:

A. 1.8N/C B. 180N/C C. 18N/C **D. 1800N/C**

8. An isolated charged point particle produces an electric field with magnitude E at a point 2m away from the charge. A point at which the field magnitude is E/4 is:

A. 1m away from the particle B. 0.5m away from the particle

C. 2m away from the particle **D. 4m away from the particle**

9. An isolated charged point particle produces an electric field with magnitude E at a point 2m away. At a point 1m from the particle the magnitude of the field is:

A. E B. 2E **C. 4E** D. E/2

10. The electric field due to a uniform distribution of charge on a spherical shell is zero:

A. everywhere B. nowhere C. only at the center of the shell **D. only inside the shell**

11. The magnitude of the force of a 400-N/C electric field on a 0.02-C point charge is:

A. 8.0N B. 8 × 10−5 N C. 8 × 10−3 N D. 0.08N

12. The purpose of Milliken's oil drop experiment was to determine:

A. the mass of an electron **B. the charge of an electron**

C. the ratio of charge to mass for an electron D. the sign of the charge on an electron

13. A charged oil drop with a mass of 2 × 10–4 kg is held suspended by a downward electric field of 300N/C. The charge on the drop is:

A. $+1.5 \times 10^{-6}\ C$ B. -1.5×10^{-6} C C. $+6.5 \times 10^{-6}$ C **D. -6.5×10^{-6} C**

14. A total charge of 6.3×10–8 C is distributed uniformly throughout a 2.7-cm radius sphere. The volume charge density is:

A. 3.7×10^{-7} C/m³ B. 6.9×10^{-6} C/m³ C. 6.9×10^{-6} C/m² **D. 7.6×10^{-4} C/m³**

15 . The flux of the electric field $(24N/C)\hat{\imath} + (30N/C)\ j + (16N/C)$ k through a 2.0m2 portion of the yz plane is:

A. 32N · m²/C B. 34N · m²/C C. 42N · m²/C **D. 48N · m²/C**

16 . A charged point particle is placed at the center of a spherical Gaussian surface. The electric flux ΦE is changed if:

A. the sphere is replaced by a cube of the same volume

B. the sphere is replaced by a cube of one-tenth the volume

C. the point charge is moved off center (but still inside the original sphere)

D. the point charge is moved to just outside the sphere

17. A conducting sphere of radius 0.01m has a charge of 1.0×10^{-9} C deposited on it. The magnitude of the electric field in N/C just outside the surface of the sphere is:

A. 0 B. 450 **C. 900** D. 4500

18. 10C of charge are placed on a spherical conducting shell. A particle with a charge of −3C is placed at the center of the cavity. The net charge on the inner surface of the shell is:

A. −7C B. −3C C. 0C **D. +3C**

19. If 500 J of work are required to carry a charged particle between two points with a potential difference of 20V, the magnitude of the charge on the particle is:

A. 0.040C **B. 12.5C** C. 20C D. None of these

20. The potential difference between two points is 100V. If a particle with a charge of 2C is transported from one of these points to the other, the magnitude of the work done is:

A. 200 J B. 100 J C. 50 J D. 100 J

21. A hollow metal sphere is charged to a potential V . The potential at its center is:

A. V B. 0 C. −V D. 2V

22. A conducting sphere has charge Q and its electric potential is V , relative to the potential far away. If the charge is doubled to 2Q, the potential is:

A. V **B. 2V** C. 4V D. V/2

23. The equi-potential surfaces associated with a charged point particles are:

A. radially outward from the particle B. vertical planes

C. horizontal planes **D. concentric spheres centered at the particle**

24. The units of capacitance are equivalent to:

A. J/C B. V/C C. J^2/C **D. C^2/J**

25. A farad is the same as a:

A. J/V B. V/J **C. C/V** D. V/C

26. A capacitor C "has a charge Q". The actual charges on its plates are:

A. Q, Q B. Q/2, Q/2 **C. Q, −Q** D. Q/2, −Q/2

27. Each plate of a capacitor stores a charge of magnitude 1mC when a 100-V potential difference is applied. The capacitance is:

A. 5 μF **B. 10 μF** C. 50 μF D. 100 μF

28. To charge a 1-F capacitor with 2C requires a potential difference of:

A. 2V B. 0.2V C. 5V D. 0.5V

29. The capacitance of a parallel-plate capacitor with plate area A and plate separation d is given by:

A. $\varepsilon_0 d/A$ B. $\varepsilon_0 d/2A$ **C. $\varepsilon_0 A/d$** D. $\varepsilon_0 A/2d$

30. The capacitance of a parallel-plate capacitor is:

A. proportional to the plate area B. proportional to the charge stored

C. independent of any material inserted between the plates D. proportional to the potential difference of the plates

31. The capacitance of a parallel-plate capacitor can be increased by:

A. increasing the charge B. decreasing the charge

C. increasing the plate separation **D. decreasing the plate separation**

32. If both the plate area and the plate separation of a parallel-plate capacitor are doubled, the capacitance is:

A. doubled B. halved **C. unchanged** D. tripled

33. If the plate area of an isolated charged parallel-plate capacitor is doubled:

A. the electric field is doubled **B. the potential difference is halved**

C. the charge on each plate is halved D. the surface charge density on each plate is doubled

34. If the plate separation of an isolated charged parallel-plate capacitor is doubled:

A. the electric field is doubled B. the potential difference is halved

C. the charge on each plate is halved **D.none of the above**

35. Pulling the plates of an isolated charged capacitor apart:

A. increases the capacitance **B. increases the potential difference**

C. does not affect the potential difference D. decreases the potential difference

36. If the charge on a parallel-plate capacitor is doubled:

A. the capacitance is halved B. the capacitance is doubled

C. the electric field is halved **D. the electric field is doubled**

37. A parallel-plate capacitor has a plate area of 0.2m2 and a plate separation of 0.1mm. To obtain an electric field of 2.0 × 106 V/m between the plates, the magnitude of the charge on each plate should be:

A. 8.9×10^{-7} C B. 1.8×10^{-6} C C. 3.5×10^{-6} C **D. 7.1×10^{-6} C**

38. A parallel-plate capacitor has a plate area of 0.2m2 and a plate separation of 0.1mm. If the charge on each plate has a magnitude of 4 × 10–6 C the potential difference across the plates is approximately:

A. 0 B. 4×10^{-2} V C. 1×10^{2} V **D. 2×10^{2} V**

39. The capacitance of a spherical capacitor with inner radius a and outer radius b is proportional to:

A. a/b B. b – a C. $b^2 - a^2$ **D. ab/(b – a)**

40. The capacitance of a single isolated spherical conductor with radius R is proportional to:

A. R B. R^2 C. 1/R D. $1/R^2$

41. Two conducting spheres have radii of R1 and R2, with R1 greater than R2. If they are far apart the capacitance is proportional to:

A. $\frac{R_1 R_2}{R_1 - R_2}$ B. $R_1^2 - R_2^2$ C. $\frac{R_1 - R_2}{R_1 R_2}$ D. R^2

42 . The capacitance of a cylindrical capacitor can be increased by:

A. decreasing both the radius of the inner cylinder and the length

B. increasing both the radius of the inner cylinder and the length

C. increasing the radius of the outer cylindrical shell and decreasing the length

D. only by decreasing the length

43. A 2μF and a 1μF capacitor are connected in series and a potential difference is applied across the combination. The 2-μF capacitor has:

A. twice the charge of the 1μF capacitor B. half the charge of the 1μF capacitor

C. twice the potential difference of the 1μF capacitor **D. half the potential difference of the 1μF capacitor**

44. Capacitors C1 and C2 are connected in parallel. The equivalent capacitance is given by:

A. $\frac{C_1C_2}{C_1+C_2}$ B. $\frac{C_1+C_2}{C_1C_2}$ **C.** $C_1 + C_2$ D. $\frac{C_1}{C_2}$

45. Capacitors C1 and C2 are connected in series. The equivalent capacitance is given by:

A. $\frac{C_1C_2}{C_1+C_2}$ B. $\frac{C_1+C_2}{C_1C_2}$ C. $C_1 + C_2$ D. $\frac{C_1}{C_2}$

46. Capacitors C_1 and C_2 are connected in series and a potential difference is applied to the Combination. If the capacitor that is equivalent to the combination has the same potential difference, then the charge on the equivalent capacitor is the same as:

A. the charge on C_1 B. the sum of the charges on C_1 and C_2

C. the difference of the charges on C_1 and C_2 D. the product of the charges on C_1 and C_2

47. Capacitors C_1 and C_2 are connected in parallel and a potential difference is applied to the combination. If the capacitor that is equivalent to the combination has the same potential difference, then the charge on the equivalent capacitor is the same as:

A. the charge on C_1 **B. the sum of the charges on C_1 and C_2**

C. the difference of the charges on C_1 and C_2 D. the product of the charges on C_1 and C_2

48. Two identical capacitors are connected in series and two, each identical to the first, are connected in parallel. The equivalent capacitance of the series connection is the equivalent capacitance of parallel connection.

A. twice B. four times C. half **D. one-fourth**

49. Two identical capacitors, each with capacitance C, are connected in parallel and the combination is connected in series to a third identical capacitor. The equivalent capacitance of this arrangement is:

A. 2C/3 B. C C. 3C/2 D. 2C

50. A 20-F capacitor is charged to 200V. Its stored energy is:

A. 4000 J B. 4 J **C. 0.4J** D. 2000 J

51. A charged capacitor stores 10C at 40V. Its stored energy is:

A. 400 J B. 4 J C. 0.2J **D. 200J**

52. The quantity (1/2)60E2 has the significance of:

A. energy/farad B. energy/coulomb C. energy **D. energy/volume**

53. Two capacitors are identical except that one is filled with air and the other with oil. Both capacitors carry the same charge. The ratio of the electric fields Eair/Eoil is:

A. between 0 and 1 B. 0 C. 1 **D. between 1 and infinity**

54. A parallel-plate capacitor, with air dielectric, is charged by a battery, after which the battery is disconnected. A slab of glass dielectric is then slowly inserted between the plates. As it is being inserted:

A. a force repels the glass out of the capacitor **B. a force attracts the glass into the capacitor**

C. no force acts on the glass D. a net charge appears on the glass

55. Two parallel-plate capacitors with the same plate separation but different capacitance are connected in parallel to a battery. Both capacitors are filled with air. The quantity that is NOT the same for both capacitors when they are fully charged is:

A. potential difference B. energy density

C. electric field between the plates D. **charges on the positive plate**

56. Two parallel-plate capacitors with the same plate area but different capacitance are connected in parallel to a battery. Both capacitors are filled with air. The quantity that is the same for both capacitors when they are fully charged is:

A. potential difference B. energy density

C. electric field between the plates D. charge on the positive plate

57. Two parallel-plate capacitors with different plate separation but the same capacitance are connected in series to a battery. Both capacitors are filled with air. The quantity that is NOT the same for both capacitors when they are fully charged is:

A. potential difference B. stored energy

C. electric field between the plates D. charge on the positive plate

58. Two parallel-plate capacitors with different capacitance but the same plate separation are connected in series to a battery. Both capacitors are filled with air. The quantity that is the same for both capacitors when they are fully charged is:

A. potential difference B. stored energy

C. energy density **D. charge on the positive plate**

SOME IMPORTANT MCQ FROM PREVIOUS EXAM ^{imp}

1. Si unit of charge is **Coulombs**.
2. Si unit of permittivity is **$C^2N^{-1}m^{-2}$**
3. Si unit of k is **$N^1m^2c^{-2}$.**
4. Ratio of magnitude of electric force in air and water between an electron and proton is K.
5. Charge on a neutron **is 0**.
6. Charge on proton is **$1.6x10^{-19}C$.**
7. Charge on electron is **$-1.6x10^{-19}C$**.
8. When the distance between the two charge particles is doubled then the force between them becomes **one fourth**.
9. Nature of electric force between the two protons is **repulsive**.
10. When the distance between the two charge particles is halved then the force becomes **four times**.
11. Charge on an atom is **0C.**
12. Torque acting on an electric dipole of dipole moment P placed at an angle 90^0 to the electric field E will be **PE**.
13. Torque acting on an electric dipole of dipole moment P placed parallel to the electric field E will be **0.**
14. Si unit of electric potential is **volt**.
15. Si unit of capacitance is **farad**.
16. The energy density of electric field E **is $\frac{1}{2}\varepsilon_0E^2$**.
17. Dielectric constant of metal **is ∞.**