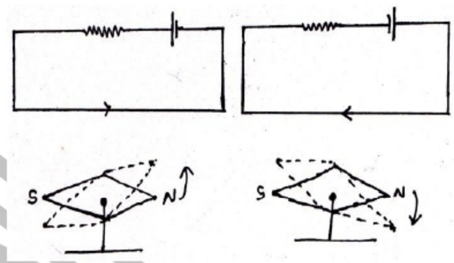
Unit – 3 Magnetic effect of current & magnetism

3(a) Magnetic effect of current

1.Electromagnetism

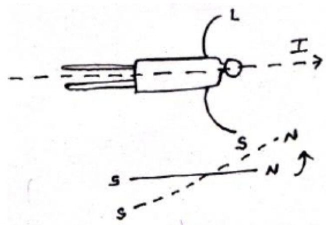
The branch of physics which deals with the study of magnetism due to electricity is called electromagnetism. The relationship between electricity& Magnetism was firstly given by Oersted which is as below



2. Oersted experiments:-

As shown in fig, Oersted found a relation between current & magnetism, when a magnetic niddle is placed parallel to direction of flow of current then the niddle suffer a deflection, when the direction of flow of current is reversed then the deflection is also reverses, the direction of deflection can be given by Ampere's swimming rule.

Ampere's swimming rule:-



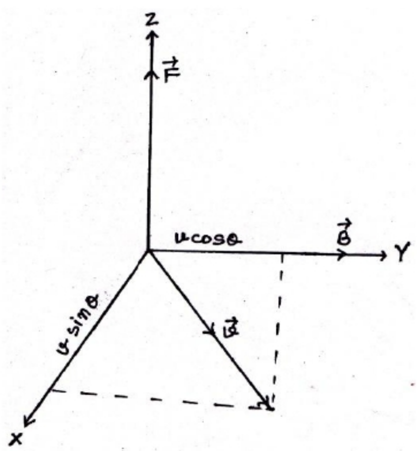
According to this rule, if we assume that a man is swimming in direction of flow of current, such that current is flowing from his feet to head then the north pole of niddle will deflect towards his left hand.

3. Magnetic field:-

The space around a magnet or a current carrying conductor in which its magnetic effect can be experienced is called magnetic field. It is a vector quantity & denoted by $\vec{B}$.

Force on a moving charge in a magnetic field or Magnetic Lorentz force.[m.imp]

Suppose a +q charge moving in a magnetic field $\vec{B}$ with velocity $\vec{v}$. Then $\vec{F}_m$ is the magnetic force experienced by the charge depends on



$$F_m \propto q \qquad (i)$$

$$F_m \propto v \sin\theta \qquad (ii)$$

$$F_m \propto B \qquad (iii)$$

Combining eq^{n}s we get $\quad F_m \propto q\,v\,B\,\text{Sin}\theta$

Or $\quad F_m = K\,q\,v\,B\,\text{Sin}\theta$

Here K = 1 so $\quad F_m = q\,v\,B\,\text{Sin}\theta$

Or in vector form $\quad \vec{F}_m = q(\vec{v} \times \vec{B})$

The direction of $\vec{F}_m$ can be given by Right hand rule. I,e. $\vec{F}_m$ *is perpendicular to plane containing both $\vec{v}$ & $\vec{B}$. Also direction of* $\vec{F}_m$ *is given by **Fleming left hand rule**.*

Definition of B:-

If $v=1$, $q = 1$ & $sin\theta = 1 \Rightarrow \theta = 90$

Then $F_m = B$

Hence magnetic field is equal to the force experienced by a unit charge moving with a unit velocity perpendicular to the direction of magnetic field at that point.

Special cases:-

(i)- If $\theta = 0^0$ or 180^0 then $\sin 0^0 = 0 \Rightarrow F_m = 0$

Hence a charge particle moving along or opposite to direction of magnetic field experience no any force.

(ii)- If $v = 0 \Rightarrow F_m = 0$

Means a rest charge in magnetic field experienced no any force.

(iii)- If $\theta = 90^0 \Rightarrow \sin 90^0 = \max \Rightarrow F_m =$ maximum.

Hence a charge particle moving perpendicular to external magnetic field experience maximum force.

Unit of magnetic field B: SI unit of B is Tesla or Weber/metre^2

As $B = \frac{F}{qvBSin\theta}$

Or $1 \text{ Tesla} = \frac{1N}{1C \times \frac{1m}{sec} \times sin 90}$

Hence magnetic field or magnetic induction is said to be one Tesla if a charge of one 1 coulomb moving perpendicular to magnetic field with a velocity of 1ms^{-1} experience a force of one Newton.

Dimensional formula: -

$$B = \frac{F}{qvBSin\theta} = \frac{[MLT^{-2}]}{[AT(LT^{-1})]} = [MA^{-1}T^{-2}]$$

In c,g,s system the unit of magnetic field is gauss G.

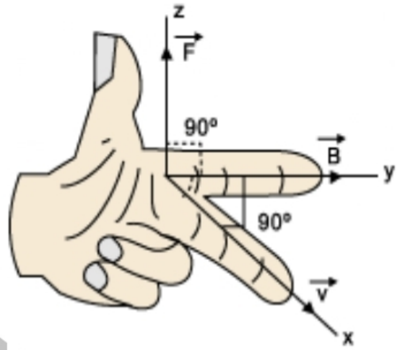
$$\mathbf{1\ T = 10^4 G}$$

4. Right hand rule or right hand screw rules:-

According to right hand rule if we curl the finger of our right hand from $\vec{v}$ to $\vec{B}$ then thumb will give the direction of $\vec{F}_m$.

5 Fleming's left hand rule:- m.imp

If we stretch the first finger, the central finger & thumb of left hand mutually perpendicular to each other, such that first finger points to the direction of magnetic field, the central finger in direction to the electric current then thumbs will give the direction of force experienced by the charge particle.



6 Biot Savart's Law:- m.imp

As we know when current passes through a conductor, then magnetic field set up around the conductor which was firstly calculated by Jean Biot & Felix Savart called Biot Savart's Law.

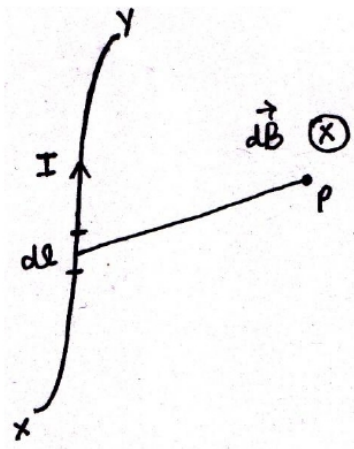
According to this law, the magnitude of magnetic field $d\vec{B}$ at a point due to current following through a conductor is depends upon, current , length of conductor, distance of point from conductor & angle between point and conductor as

$$dB \propto I \quad --(i)$$

$$dB \propto dl \quad --(ii)$$

$$dB \propto sin\theta \quad --(iii)$$

$$dB \propto \frac{1}{r^2} \quad --(iv)$$



Combining all eq^{n}s we get $dB \propto \frac{IdlSin\theta}{r^2}$

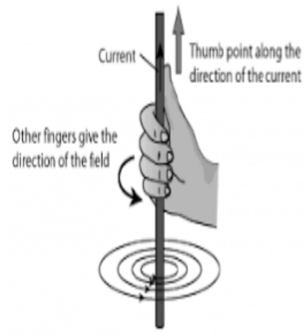
Or $dB = \frac{K\,IdlSin\,\theta}{r^2}$

Where $K = \frac{\mu_0}{4\pi} = 10^{-7}$ TmA^{-1}

Here u_0 called *permeability of free space*.

So Biot Savart's law become $dB = \frac{\mu_0}{4\pi} \frac{K\,IdlSin\,\theta}{r^2}$

In vector form $\overrightarrow{dB} = \frac{\mu_0}{4\pi} \frac{I\,\vec{dl} \times \vec{r}}{r^3}$



The direction of dB can be given by right hand rule. In present case the direction of $\overrightarrow{dB}$ is perpendicular to plane of paper containing both $\vec{dl}$ & $\vec{r}$ in inward direction; if P point lies in L.H.S then in outwards direction.

Special cases:-

(i) If $\theta = 0^0,\ Sin\theta = 0 \Rightarrow dB = 0$

(ii) If $\theta = 90^0,\ Sin90^0 = 1 = max \Rightarrow dB = maximum$

I,e. *magnetic field at a point perpendicular to current element is maximum*

- ✓ This law is applicable for infinite small conductor.
- ✓ This law is similar to coulomb's law in electrostatics.
- ✓ This law is difficult to verify experimentally because it is difficult to construct infinite small conductor.

7 Biot Savart's law V/s Coulomb's Law:-

As according to Coulomb's Law $dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}$

& from Biot Savart's law $dB = \frac{\mu_0}{4\pi}\frac{IdlSin\,\theta}{r^2}$

On comparing above eq^{n}s we can give similarity & dissimilarity between both.

Similarity

1. Both the law obey inverse square law ($\alpha \frac{1}{r^2}$)
2. Both the law obeys principle of Superposition.

Dissimilarity:-

1. The source of magnetic field is current while source of electric field is charged.
2. Electric field exists due to both rest & motion of charge while magnetic field exists due to motion of charge.
3. The magnetic field is angle dependent while electric field is not.

8 Magnetic field due to a long straight Conductor carrying current:- m.imp

Suppose a point P having a distance a from the straight conductor carrying current I, then according to Biot savart Law, the magnetic field $\vec{dB}$ at P due to small current element $\vec{Idl}$, r distance apart is

$$dB = \frac{\mu_0}{4\pi}\frac{IdlSin\,\theta}{r^2} \quad \text{--(1)}$$

Now from Δ COP $\theta + \emptyset = 90^0 \Rightarrow \theta = 90 - \emptyset$

Or $sin\theta = sin(90 - \emptyset) = cos\emptyset$ i

Also $cos\emptyset = \frac{a}{r} \Rightarrow r = \frac{a}{cos\,\emptyset} = a\,sec\,\emptyset$ ii

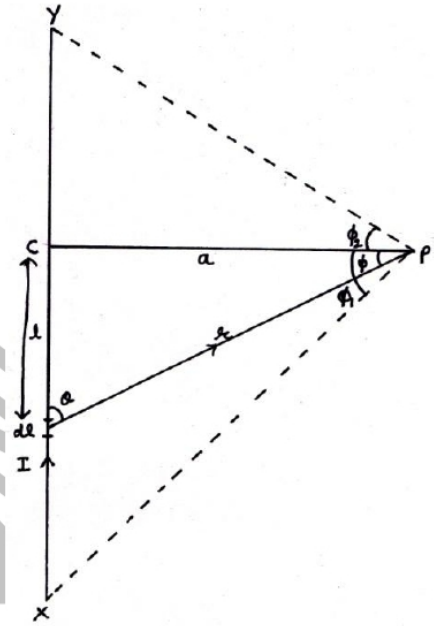
As $\tan\emptyset = \frac{l}{a} \Rightarrow l = a\tan\emptyset$

Differentiating both sides $dl = a \sec^2 \emptyset . d\emptyset$ iii

Using all values in eq^ns (1) we get

$$dB = \frac{\mu_0}{4\pi} \frac{I.a \sec^2\emptyset.d\emptyset.\cos\emptyset}{a^2 \sec^2\emptyset.}$$

Or $$dB = \frac{\mu_0}{4\pi} \frac{I \cos\emptyset . d\emptyset}{a}$$



To calculate complete magnetic field integrating both sides we get $B = \frac{\mu_0}{4\pi} \frac{I}{a} \int_{-\emptyset_1}^{\emptyset_2} \cos\emptyset . d\emptyset$

$$B = \frac{\mu_0}{4\pi} \frac{I}{a} [\text{Sin}\emptyset]_{-\emptyset_1}^{\emptyset_2}$$

$$B = \frac{\mu_0}{4\pi} \frac{I}{a} [\text{Sin}\emptyset_2 - \text{Sin}(-\emptyset_1)]$$

Or $$\mathbf{B = \frac{\mu_0}{4\pi} \frac{I}{a} [Sin\emptyset_1 + Sin\emptyset_2]}$$

The direction of B can be given by *Right hand rule.*

Special cases:-

(i) If the conductor is infinite long & point P lies near to the dipole then

$$B = \frac{\mu_0}{4\pi} \frac{I}{a} [\sin 90^0 + \sin 90^0] \quad \text{Or} \quad \mathbf{B = \frac{\mu_0}{4\pi} \frac{2I}{a} = \frac{\mu_0 I}{2\pi a}}$$

(ii) If the conductor is infinite long but P lies near end y then $\emptyset_1 = 90$, $\emptyset_2 = 0$

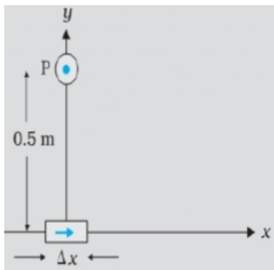
$$B = \frac{uI}{4\pi a} [\sin 90 + \sin 0] = \frac{\mu_0}{4\pi} \frac{I}{a}$$

Example1. A straight wire of mass 200 g and length 1.5 m carries a current of 2 A. It is suspended in mid-air by a uniform horizontal magnetic field B (Fig. 4.3). What is the magnitude of the magnetic field?

Solution we find that there is an upward force F, of magnitude $I\,lB$, for mid-air suspension, this must be balanced by the force due to gravity: $m\,g = I\,lB \Rightarrow B = \frac{mg}{Il} = \frac{0.2 \times 9.8}{2 \times 1.5}$ =0.65 T

Note that it would have been sufficient to specify $\frac{m}{l}$, the mass per unit length of the wire. The earth's magnetic field is approximately 4×10^{-5}T and we have ignored it.

Example2. An element $\hat{x}\Delta = \Delta\, l\, \hat{\imath}$ is placed at the origin and carries a large current I = 10 A. What is the magnetic field on the y-axis at a distance of 0.5 m. Δx = 1 cm.



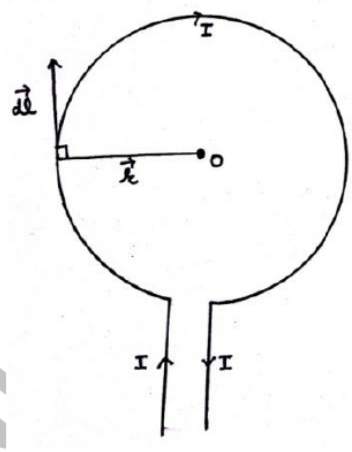
Solution $dB = \frac{\mu_0}{4\pi} \frac{Idl\text{Sin}\,\theta}{r^2}$ here $dl = 10^{-2}m$ $I = 10\,A$, $r = 0.5\,m = y$, $\frac{\mu_0}{4\pi} = 10^{-7}\,TmA^{-1}$,

$\theta = 90°$; $\sin\theta = 1 \Rightarrow$ $dB = \frac{10^{-7} \times 10 \times 10^{-2} \times 1}{(0.5)^2} = 4 \times 10^{-8}\ T$

The direction of the field is in the +z-direction. As $\vec{dl} \times \vec{r} = \Delta x\hat{\imath} \times y\hat{\jmath} = y\Delta x(\hat{\imath} \times \hat{\jmath}) = y\Delta x\hat{k}$

9 Magnetic field At the Center Of The Circular Coil Carrying current:- m.imp

According to Biot Savart Law, the magnetic field at the center of circular coil carrying current can be given as $dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$ --(1)



Here $sin\theta$ is angle between current element Idl & r. As radius is always perpendicular to tangent so $\theta = 90^0 \Rightarrow \sin 90 = 1$

$$dB = \frac{\mu_0}{4\pi}\frac{Idl}{r^2}$$

To calculate complete magnetic field, integrate both sides

$$\Rightarrow B = \frac{\mu_0}{4\pi}\frac{I}{r^2}\int dl$$

Here $\int dl = 2\pi r$ circumference of circle

$$\Rightarrow B = \frac{\mu_0}{4\pi}\frac{I}{r^2} \times 2\pi r$$

Or $B = \frac{\mu_0}{4\pi}.\frac{2\pi I}{r}$

If the circular coil contains n turns then $B = \frac{\mu_0}{4\pi}\frac{2n\pi I}{r}$

Or $\mathbf{B = \frac{\mu_0 nI}{2r}}$

Here the direction of magnetic field can be given by right hand rule.

Example3. Consider a tightly wound 100 turns coil of radius 10 cm, carrying a current of 1 A. What is the magnitude of the magnetic field at the centre of the coil?

Solution Since the coil is tightly wound, we may take each circular element to have the same radius r = 10 cm = 0.1 m. The number of turns n = 100. The magnitude of the magnetic field is, $B = \frac{\mu_0 nI}{2r} = \frac{4\times3.14\times10^{-7}\times100\times1}{2\times0.1} = 6.28\times10^{-4}T$

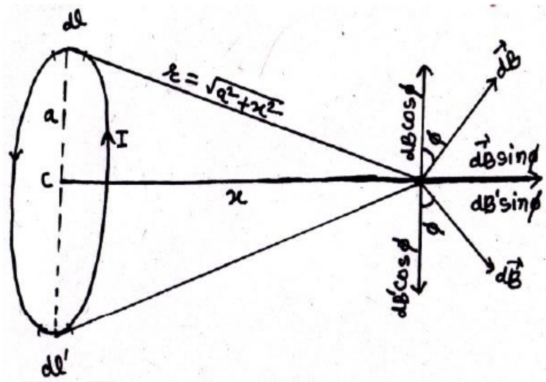
10 Magnetic field at a point on the axis of a circular coil carrying current:-

Suppose a point P on the axis of a Circular coil carrying current I. Suppose two small current element dl & dl' on coil. Then the Magnetic field due to these current Elements at P is dB & dB'. As shown in fig.

Clearly dB & dB' are equal. Resolving dB & dB' into rectangular components, we see that $dBcos\emptyset$ & $dB'cos\emptyset$ are equal & opposite so they are cancelled out. So the total magnetic induction at P due to current through the whole circular coil is given

$$B = \int dB \sin\emptyset \quad \text{–(i)}$$

From Ampere circuital law, the magnetic induction due to dl at P is $dB = \frac{\mu_0}{4\pi}\frac{IdlSin\,\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{IdlSin\,90^0}{(a^2+x^2)}$



Here θ is angle between r & dl so 90^0.

So $dB = \frac{\mu_0}{4\pi}\frac{Idl}{(a^2+x^2)}$

Using in (i) $B = \int \frac{\mu_0}{4\pi}\frac{Idl}{(a^2+x^2)} Sin\emptyset$

Here $\sin\emptyset = \frac{a}{\sqrt{a^2+x^2}}$ & $\int dl = 2\pi a$

$\Rightarrow B = \frac{\mu_0}{4\pi}\frac{I}{(a^2+x^2)}.\frac{a}{\sqrt{a^2+x^2}}.2\pi a$

Or $\boldsymbol{B = \frac{\mu_0 I\, 2\pi a^2}{4\pi(a^2+x^2)^{\frac{3}{2}}}}$

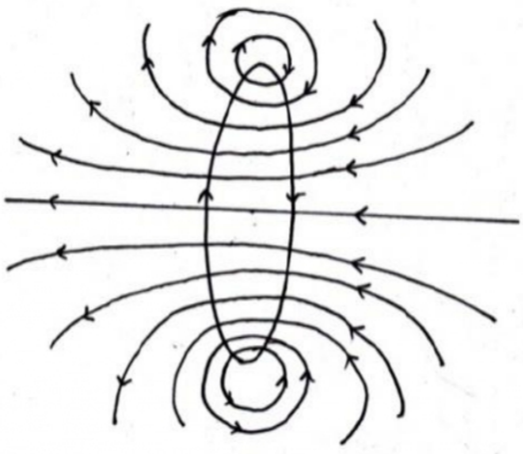
If there are n turn in the coil then $B = \frac{\mu_0 2n\pi I\, a^2}{4\pi(a^2+x^2)^{\frac{3}{2}}}$

- ✓ at center $x = 0$ $\Rightarrow$ $B = \frac{\mu_0 2n\pi I\, a^2}{4\pi(a^2)^{\frac{3}{2}}} = \frac{\mu_0}{4\pi}\frac{2n\pi I}{a}$
- ✓ If P point lies far away from the coil then $x \gg a$ so a can be neglected
- ✓ $\Rightarrow B = \frac{\mu_0 2n\pi I\, a^2}{4\pi(x^2)^{\frac{3}{2}}} = \frac{\mu_0 2n\pi I\, a^2}{4\pi(x)^3}$
- ✓ If P point lies at a distance equal to radius of the coil I,e. r = a we have

$\Rightarrow B = \frac{\mu_0 2n\pi I\, a^2}{4\pi(a^2+a^2)^{\frac{3}{2}}} = \frac{\mu_0 2n\pi I\, a^2}{4\pi\sqrt{2}\, a^3} = \frac{\mu_0\sqrt{2}\, n\pi\, I}{4\pi a}$

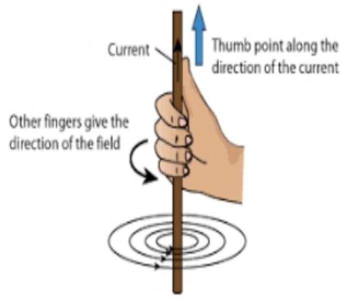
11 The direction of magnet field due to circular coil:- m.imp

As we can see the magnetic lines form Close loop at the end of the circular coil & straight line at the center of the loop. The direction of these magnetic lines can be given by Right hand rule.



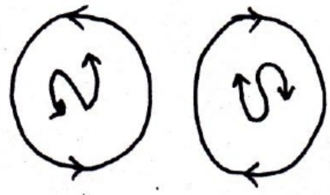
12 Right hand rule:-

Suppose the current is flowing through a circular conductor, if we imagine the fingers of the right hand curling in the direction of current, then the thumb will point in the direction of magnetic field.



13 Clock rule:-

According to clock rule if current moves in anti clock wise direction, the upper face of loop or coil is behave as north pole & when current moves in clock wise direction, then upper face behave as south pole.



14 Ampere's circuital law:- m.imp

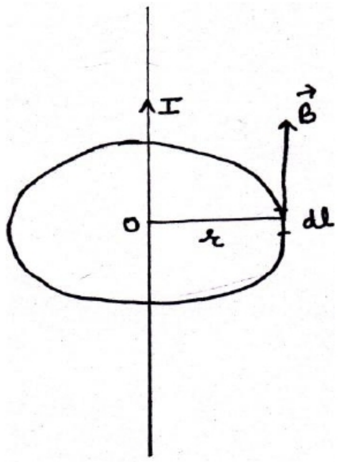
According to Ampere's circuital law, the line integral of magnetic field around a closed circuit is equal to μ_0 times the total current flowing through the circuit.

I,e. $\oint \vec{B}.\vec{dl} = \mu_0 I$

Proof: -

Suppose a Ampere's surface around a straight conductor carrying current I. Now the magnet field at P due to current I in conductor can be given by

$$B = \frac{\mu_0 I}{2\pi r}$$



Now the line integral of $\vec{B}$ is $\oint \vec{B}.\vec{dl} = \frac{\mu_0 I}{2\pi r} \oint dl$

Here $\oint dl = 2\pi r$= circumference of the circle

So $\oint \vec{B}.\vec{dl} = \frac{\mu_0 I}{2\pi r} \times 2\pi r$

Or $\oint \vec{B}.\vec{dl} = \mu_0 I$

This is required expression for Ampere circuital law.

- ✓ *Ampere's circular law holds for DC or steady current which does not change with time.*
- ✓ *Ampere's circuital law for magnetic field is analogy to Gauss law in electrostatics.*
- ✓ *Biot Savart's law & Ampere's circuital law are equivalent in same sense as Coulombs Law & Gauss theorem in electrostatics.*
- ✓ *For a loop that is not connected to the current* $B.dl = 0$

15 Applications of Ampere's Circuital law [m.imp]

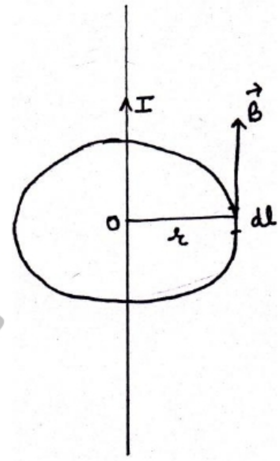
(a) Magnetic field due to an infinite long current carrying wire:-

Suppose an infinite long straight conductor carrying current I. Now we have to calculate magnetic field at P on the Amperean loop of radius r. Consider small length dl at loop, then magnetic field B at dl can be given by Ampere's circuital law as

$$\oint \vec{B}.\vec{dl} = \mu_0 I$$

Or $$\oint B.dl\cos 0^0 = \mu_0 I$$

Or $$B\oint dl = \mu_0 I$$



Here $\oint dl$ = circumference of circle = $2\pi r$

$$\Rightarrow B \times 2\pi r = \mu_0 I$$

$$\Rightarrow \mathbf{B} = \frac{\mu_0 I}{2\pi r}$$

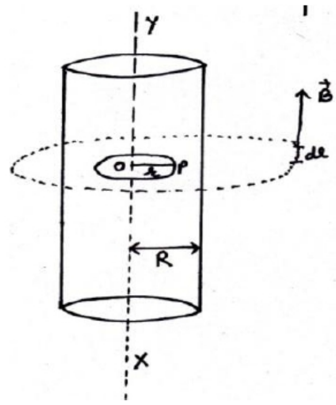
(b) Magnetic field due to current through a very long circular cylinder (not directly in syllabus)

Suppose a cylinder of radius R carrying current I through its axis. Then the magnetic field around the cylinder will be in circular form.

1. *If the P point lies outside the cylinder Then from Ampere circuital law.*

$$\oint B.dl = \oint B.dl\cos 0^0 = \mu_0 I$$

$$\Rightarrow B\oint dl = \mu_0 I$$



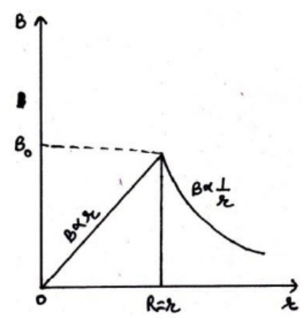
Here $\oint dl = 2\pi r$ = circumference of circle

$$\Rightarrow B \times 2\pi r = \mu_0 I$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi r}$$ I,e. $(B \propto \frac{1}{r})$

2. *If P point lies inside the cylinder than*

$$I' = \frac{I}{\pi R^2} x\, \pi r^2 = \frac{Ir^2}{R^2}$$



Now applying Ampere's circuital law

$$\oint B.dl = \mu_0 u_r I'$$

$$B \times 2\pi r = \mu_0 u_r \frac{Ir^2}{R^2}$$

$$\text{Or} \quad B = \frac{\mu_0 u_r Ir}{2\pi R^2} \qquad I, e. B \propto r$$

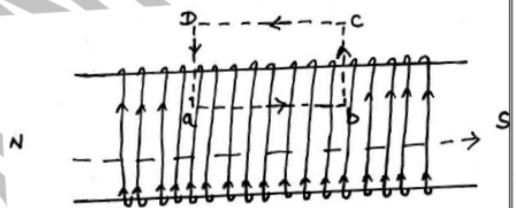
The relation between B & r is as shown in fig.

16c Solenoid:- m.imp

Solenoid means an insulated copper wire wounded closely in the form of a helix. The length of the solenoid is very large as compared to its diameter.

Calculation of magnetic field inside a long straight solenoid:-

Suppose a rectangular Amperean loop $abcda$ of length l. If N is the number of turns in the length l, then total current through the loop equal to NI.



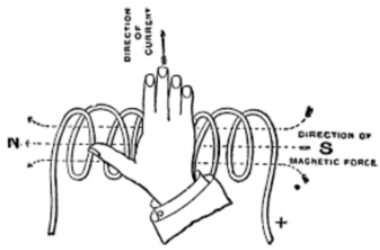
Thus according to Amperes law

$$\oint \vec{B}.\vec{dl} = \mu_0 NI \qquad \text{--(1)}$$

Or $\quad \int_{abcda} \vec{B}.\vec{dl} = \int_a^b \vec{B}.\vec{dl} + \int_b^c \vec{B}.\vec{dl} + \int_c^d \vec{B}.\vec{dl} + \int_d^a \vec{B}.\vec{dl}$

Here $\quad \int_b^c \vec{B}.\vec{dl} = \int_d^a \vec{B}.\vec{dl} = \int \vec{B}.\vec{dl}\cos 90^0 = 0$

Also $\quad \int_c^D \vec{B}.\vec{dl} = 0$ because CD lies outside the solenoid.



so $\quad \oint \vec{B}.\vec{dl} = \int_a^b \vec{B}.\vec{dl} = Bl \qquad$ ..(2)

Comparing eq^n 1 & 2 we get $\quad Bl = \mu_0 NI$

Or $\quad B = \frac{\mu_0 NI}{l}$

Or $\quad \mathbf{B = \mu_0 nI}$

Where n is the number of turns per unit length.

Example4. A solenoid of length 0.5 m has a radius of 1 cm and is made up of 500 turns. It carries a current of 5 A. What is the magnitude of the magnetic field inside the solenoid?

Solution The number of turns per unit length is, $n = \frac{500}{0.5} = 1000$ turns/m

The length $l = 0.5\ m$ and radius $r = 0.01\ m$ Thus, $\frac{l}{a} = 50\ i.e., l >> a.$

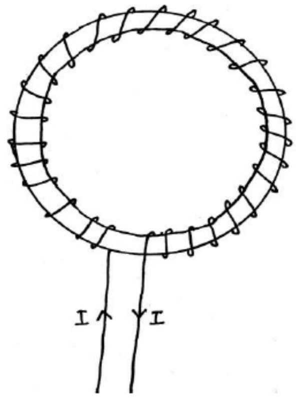
Hence, we can use the long solenoid formula, namely, B = $\mu_0 nI = 4\pi \times 10^{-7} \times 10^3 \times = 6.28 \times 10^{-3} T$

16d Toroid or Toroidal Solenoid:- m.imp

A solenoid bent into the form of a closed ring is called a toroidal solenoid.

Let us consider a toroid having N number of turn equally spaced & let I is the amount of current flowing through them.

(a) *Magnetic field at a point inside the core of the solenoid:-*



From Ampere circuital law $\oint \vec{B}.\vec{dl} = \mu_0 I$

From N number of turns $\oint \vec{B}.\vec{dl} = \mu_0 NI$

Here $\oint dl = 2\pi r$ ⇨ $B \times 2\pi r = \mu_0 NI$

⇨ $B = \frac{\mu_0 NI}{2\pi r}$

Or $\mathbf{B = \mu_0 nI}$

Where $n = \frac{N}{2\pi r}$ = number of turns per unit length, Thus magnetic is same as that of solenoid.

(b) Magnetic field inside the toroid:-

As inside a hollow conductor I = 0 ⇨ $\oint \vec{B}.\vec{dl} = \mu_0(0) = 0$ ⇨ B=0

Thus Magnetic field inside the toroid is zero

(c) Magnetic field outside the solenoid.

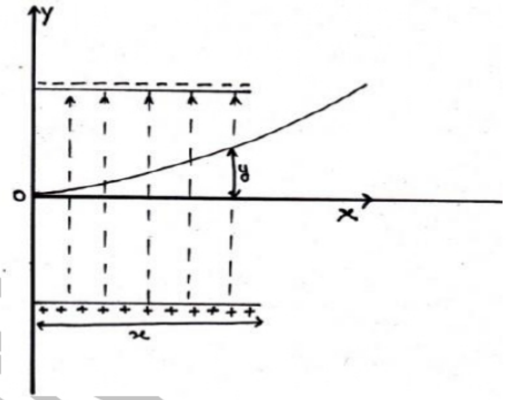
For any point outside the tortoid, the current threading the loop L' through current at the point is zero so B = 0 ⇨ $\oint \vec{B}.\vec{dl} = 0$

This is the condition for ideal toroid in which the turns of the wire are very closely spaced & the magnetic field is within the toroid. For outside B = 0. I,e. magnetic field does not comes outside as in solenoid.

3(b) Force on a charge particle in Electric & Magnetic field

17 Motion of a charge particle in a uniform electric field:-

Let us consider a charge particle + q moving with v velocity in the electric field then charge particle will experience a electric force.



As $\vec{F} = q\vec{E}$ -(1)

Due to this electric force charge particle starts accelerating with a force.

$\vec{F} = m\vec{a}$ -(2)

Comparing eq^n (1) & (2) we get $m\vec{a} = q\vec{E}$

$\vec{a} = \frac{q\vec{E}}{m}$ -(3)

Now let the time taken by the charge particle to move x distance is $t = \frac{x}{v}$

Similarly the distance travelled along y axis is $y = u_y t + \frac{1}{2} a_y t^2$

Here $u_y = 0$ also $\vec{a} = \frac{q\vec{E}}{m}$ & $t = \frac{x}{v}$

So $y = \frac{1}{2}\frac{q\vec{E}}{m}(\frac{x}{v})^2 = \frac{1}{2}\frac{q\vec{E}x^2}{mv^2}$

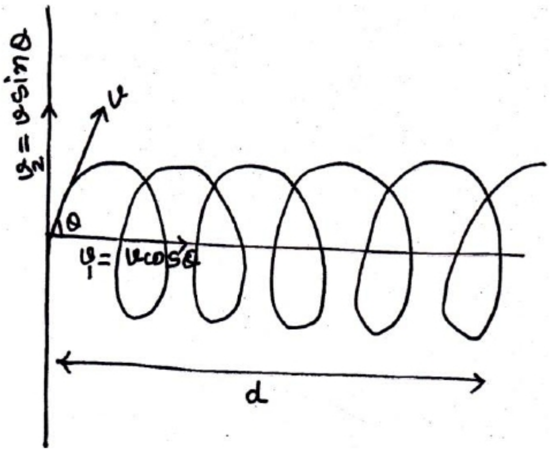
Or $y = kx^2$ where $k = \frac{1q\vec{E}}{2mv^2}$

or $y \propto x^2$

Which is an eq^n of parabola, hence we can say that particle will follow ***parabolic path.***

18 Motion of charge particle in uniform magnetic field:-

Let us suppose a charge particle +q enter in a magnetic field B at origin O with a velocity v, making an angle θ with magnetic field. Now resolving v into two rectangular components we have $v_1 = v\cos\theta$ along B and $v_2 = v \sin\theta$ along perpendicular to magnetic field, the force on the charge particle due to v_2 is



$\vec{F_2} = q\,(\vec{v_2} \times \vec{B})$

Or $F = q\,v_2 B\sin 90^0$

Or $F = qv_2B$

The direction of this force can be obtained by R.H.R & is perpendicular to plane containing $\vec{v_2}$ & $\vec{B}$. Here motion of the charge particle is perpendicular to applied force so work done by this force is zero.

Hence this force provides no motion to the charge particle but gives the circular motion. i,e. centripetal force. So we may write $qv_2B = \frac{mv_2^2}{r}$

Or $v_2 = \frac{qBr}{m}$

The angular velocity of the particle can be given as $\omega = \frac{v_2}{r} = \frac{qBr}{mr} = \frac{qB}{m}$

The frequency of rotation of the particle in magnetic field will Becomes $\nu = \frac{\omega}{2\pi}$

Or $\nu = \frac{qB}{2\pi m}$

So the time period $T = \frac{1}{\nu} = \frac{2\pi m}{qB}$

For component $v_1 = v\cos\theta$ the force is

$$\vec{F_1} = q(\vec{v_1} \times \vec{B}) = q\, v_1 B \sin 0^0 = 0$$

Here there is no any force due to v_1 so the charge particle moves linearly due to component v_1.

*Thus by the combined effect of v_1 & v_2 charge particle move linear as well as circular so its path becomes **helical**.*

The total distance travelled by the particle is $d = v_1 T = \frac{v\cos\theta \,.\, 2\pi m}{qB}$

Example5. what is the radius of the path of an electron (mass 9×10^{-31} kg and charge 1.6×10^{-19} C) moving at a speed of 3×10^7 m/s in a magnetic field of 6×10^{-4} T perpendicular to it? What is its frequency? Calculate its energy in keV.

(1 eV = 1.6×10^{-19} J).

Solution as $r = \frac{mv}{qB} = \frac{(9 \times 10^{-31}\ kg \times 3 \times 10^7\ m\ s^{-1})}{(1.6 \times 10^{-19}\ C \times 6 \times 10^{-4}\ T)} = 26 \times 10^{-2}$ m = 26 cm

$$\nu = \frac{v}{(2\pi r)} = 2 \times 10^6\ s^{-1} = 2 \times 10^6\ \text{Hz} = 2\ \text{MHz}.$$

$$E = \frac{1}{2}mv^2 = \frac{1}{2} \times 9 \times 10^{-31} kg \times 9 \times 10^{14} m^2/s^2 = 40.5 \times 10^{-17} J \approx 4 \times 10^{-16} J = 2.5\, keV.$$

19 Lorentz force $^{m.imp}$

The total force experienced by a charge particle in electric & magnetic field both is called Lorentz force. The force experienced by charge particle in electric field is

$$\vec{F_e} = q\vec{E} \quad --(1)$$

& the force experienced in magnetic field is

$$\vec{F_m} = q(\vec{v} \times \vec{B}) \quad –(2)$$

Thus the total force (Lorenz force) experienced by charge particle is

$$\vec{F} = \vec{F_e} + \vec{F_m}$$

Or $\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$

Or $\mathbf{F} = q\{\vec{\boldsymbol{E}} + (\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}})\}$

Special cases:-

1. When all $\vec{E}$,$\vec{v}$ & $\vec{B}$ are collinear

   In this case $\overrightarrow{F_m}$ = 0 because $vBsin\,0 = 0$ so the total force on the charge particle will be $\vec{F} = \vec{F_e}$ = q$\vec{E}$

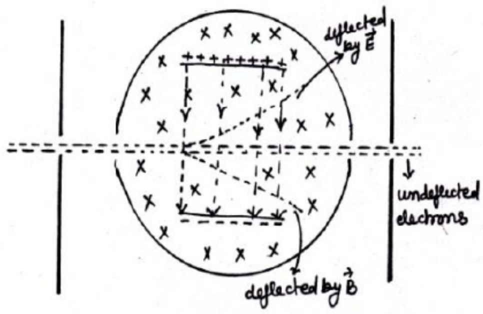
2. When $\vec{E}$,$\vec{v}$ & $\vec{B}$ are mutually perpendiculars to each other. In this case $\vec{E}$ & $\vec{B}$ are such that

$$\vec{F} = \vec{F_e} + \overrightarrow{F_m} = 0, \text{ then } \vec{a} = \frac{\vec{F}}{m} = 0$$

It means the particle will pass through the field without any change in velocity. The concept is use in velocity Selector to obtain a charged beam having a definite velocity.

20 Motion of charge particle in perpendicular magnetic & Electric field:- Or Velocity Selector:-

In a Velocity Selector there are both electric field and magnetic field are perpendicular to each other. They are also perpendicular to direction of motions of electrons. The electrons which have equal and opposite electric and magnetic field may pass through the velocity selector. The velocity of electrons in this case may be given as



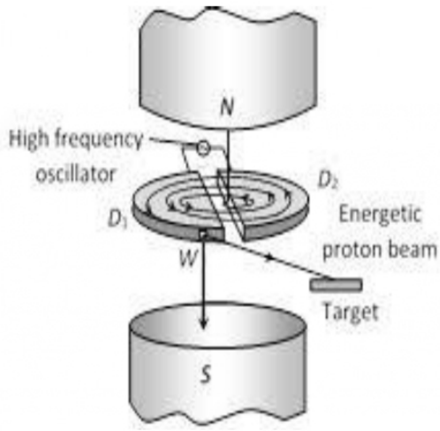
$$eE = evB \Rightarrow v = \frac{E}{B}$$

21 CYCLOTRON m.imp

It is a device which is used to accelerate the charge particle like protons, deuterons, α particles etc to a very high energy.

Principle:-

A cyclotron is based on the principle that a charge particle can be accelerated to a very high energy with help of a small electric field, and with the help of a perpendicular magnetic field, which provide the circular motion to the charge particle.

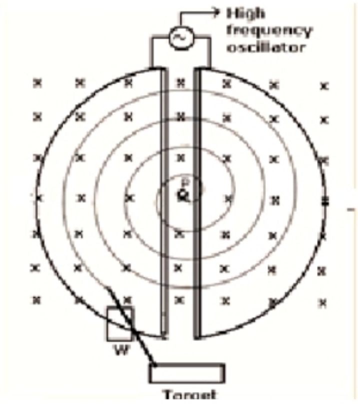


Construction

A cyclotron is consisting of two hollow small metallic Dees D_1 & D_2. A magnetic field is applied perpendicular to the plane of these Dees. These Dees are connected with a high frequency oscillator. A window W is provided in Dees D_1.

Working:-

Suppose a positive charge particle enters into the gap between the Dees & find the Dee D_1 at positive potential and D_2 at negative potential. Then charge particle enter into Dee D_2 & due to perpendicular magnetic field the path of the charge particle become circular, due to this the charge particle again comes into the gap. Now at this time the Dee D_1 becomes negative & D_2 becomes positive. So the charge particle enters into D_1 with increased velocity. This process is carried out again & again till the charge particle acquire sufficient amount of kinetic energy to hit a target.



Theory:-

Let a charge particle of mass m, charge q, and enter into magnetic field. The necessary centripetal force provided by magnetic field is $qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB}$

So the time period of revaluation of charge particle will be $T = \frac{2\pi r}{v} = \frac{2\pi}{v}\left(\frac{mv}{qB}\right) = \frac{2\pi m}{qB}$

& the frequency of revolution will be Cyclotron frequency = $f_c = \frac{1}{T} = \frac{qB}{2\pi m}$ = constant

Cyclotron frequency is independent on velocity of the charge particles & radius of the orbit.

Maximum kinetic energy acquired by the charge particle:-

If v_{max} is the maximum velocity of the charge particle & r_0 is the radius of the Dees then

$$\frac{mv_{max}^2}{R0} = qv_{max}B \quad or \quad v_{max} = \frac{qBr_0}{m}$$

So the maximum K.E of charge particle is $K.E_{max} = \frac{1}{2}mv_{max}^2 = \frac{1}{2}\frac{q^2B^2r_0^2}{m}$

Limitations of the cyclotron:-

1. According to Einstein's theory of Relativity $\boldsymbol{m = \frac{m_0}{\sqrt{1-v^2/c^2}}}$

   Here we can see that with increase in the velocity of the particle mass increases. This will decrease the cyclotron frequency & ion cannot be accelerated properly. This Limitation is overcome by using ***synchrotron.***
2. Electron cannot be accelerated in cyclotron because its velocity increases very fastly & it cannot be accelerated due to unmatching with the frequency of oscillator.
3. Neutron being electrically neutral cannot be accelerated in cyclotron.

Uses:- It is used in nuclear reactions, atomic reactor, for synthesis new materials, in hospitals for diagnosis and treatment.

Example6. a cyclotron's oscillator frequency is 10 MHz. what should be the operating magnetic field for accelerating protons? If the radius of its 'Dees' is 60 cm, what is the kinetic energy (in M eV) of the proton beam produced by the accelerator. (e =1.60 × 10^{-19} C, mp = 1.67 × 10^{-27} kg, 1 MeV = 1.6 × 10^{-13} J).

Solution The oscillator frequency should be same as proton's cyclotron frequency. We have

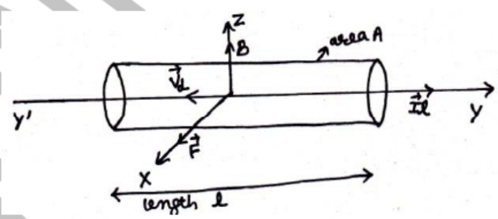
$$B = \frac{2\pi\, mv}{q} = \frac{6.3 \times 1.67 \times 10^{-27} \times 10^{7}}{(1.6 \times 10^{-19})} = 0.66\, T$$

Final velocity of protons is $v = r \times 2\pi\, v = 0.6\, m \times 6.3 \times 10^{7} = 3.78 \times 10^{7}\ m/s.$

$$E = \frac{1}{2}\, mv^2 = \frac{1.67 \times 10^{-27} \times 14.3 \times 10^{14}}{2 \times 1.6 \times 10^{-13}} = 7\, MeV.$$

22 Force on a current carrying conductor placed in magnetic field. m.imp

Consider a conductor PQ of length l and area A. Suppose current flows in the conductor along + ve y axis & magnetic field is applied along +ve z axis. So the velocity $\overrightarrow{v_d}$ of electrons will be along-ve y axis. The magnetic force experienced by the electron may be given as



$$\vec{f} = -e\,(\overrightarrow{v_d} \times \vec{B})$$

If there are n numbers of electrons per unit volume then total numbers of electrons are $N = nAl$

So the total force experienced by the electrons is $\vec{F} = N\vec{f} = -nAle\,(\overrightarrow{v_d} \times \vec{B})$ (1)

As we know $I = Ane\, v_d$

Or $\vec{Il} = -Ane\,\overrightarrow{v_d}l$ (As $\vec{Il}$ & $\overrightarrow{v_d}$ are in opposite direction so -ve sign is taken,)

Now from eq^n (1) $\boldsymbol{\vec{F} = (\vec{Il} \times \vec{B})}$

Or $\boldsymbol{F = IlBsin\theta}$ where θ is the direction between $\vec{Il}$ & $\vec{B}$.

Direction of force:-

The direction of force is given by Fleming's left hand rule or Right hand thumb rule.

Special cases:-

1. If θ = 0° or 180° then sin0 = 0 ⇒ F = 0

Hence conductor placed parallel to direction of magnetic field does not experience any force.

2. If $\theta =$ 90° ⇒ sin 90 = 1 ⇒ $F = IlB(max)$

When conductor is placed perpendicular to magnetic field it will experience maximum force.

Example7. the horizontal component of the earth's magnetic field at a certain place is 3.0 $\times 10^{-5}$T and the direction of the field is from the geographic south to the geographic north. A very long straight conductor is carrying a steady current of 1A. What is the force per unit length on it when it is placed on a horizontal table and the direction of the current is (a) east to west; (b) south to north?

Solution $F = \vec{I}\vec{l} \times \vec{B} = IlB\,sin\theta$

The force per unit length is $f = \frac{F}{l} = I\,B\,sin\theta$

(a)When the current is flowing from east to west, θ = 90° Hence, $f = I\,B = 1 \times 3 \times 10^{-5} = 3 \times 10^{-5} N\,m^{-1}$

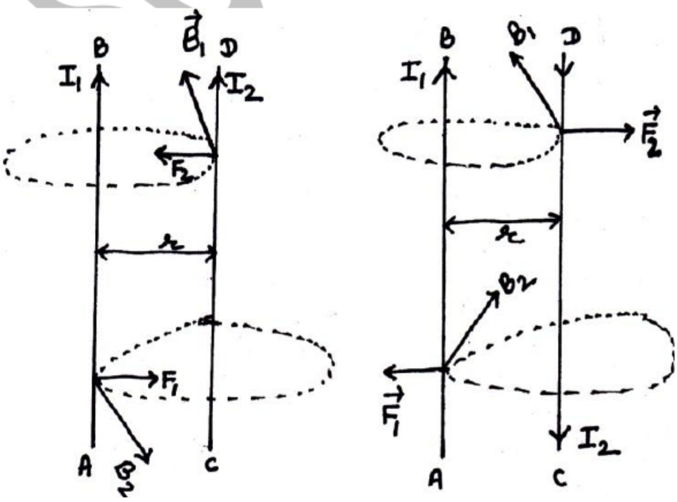
This is larger than the value 2×10–7 Nm–1 quoted in the definition of the ampere. Hence it is important to eliminate the effect of the earth's magnetic field and other stray fields while standardizing the ampere. The direction of the force is downwards. This direction may be obtained by the directional property of cross product of vectors.

(b) When the current is flowing from south to north, $\theta = 0^o$ so $f = 0$ hence there is no force on the conductor

23 Expression for the force between two Parallel current carrying wire m.imp

Consider two long parallel wires AB & CD carrying current I_1 , & I_2. Let r be the separation between them. The magnetic field produced by current I_1 at any point on wire CD is

$$B_1 = \frac{\mu_0 I_1}{2\pi r} = \frac{\mu_0}{4\pi}\frac{2I_1}{r}$$



This field acts perpendicular to the wire CD and points into the plane of the paper .It exerts a force given as

$$F_2 = B_1 I_2 l = \frac{\mu_0}{4\pi}\frac{2I_1 I_2 l}{r}$$

According to Fleming's left hand rule, this force acts at right angle to CD, toward AB in the plane of the paper.

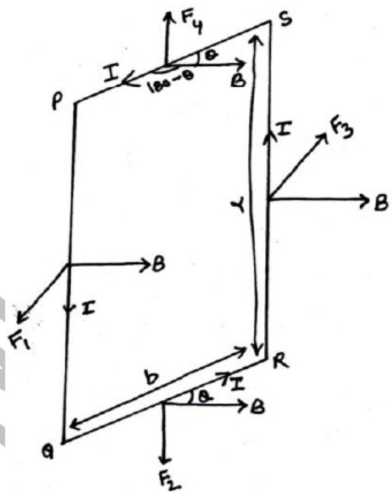
Similarly an equal force is exerted on the wire AB by the field of wire CD.

As $F_1 = B_2 I_1 l = \frac{\mu_0}{4\pi}\frac{2I_1 I_2 l}{r}$

Again according to Fleming's left hand rule, this force acts at right angle to AB, toward CD in the plane of the paper. *Thus when the current in the two wires are in same direction, the force is attractive & when current are in opposite direction, force is repulsive.*

24 Torque experienced by a current loop in a uniform magnetic field:- m.imp

.Let us consider a current carrying rectangular coil of sides l & b such that area $A = lb$. Suppose this coil is placed at an angle θ with magnetic field B. Now as we know that when a current carrying conductor placed in magnetic field experience a force,



so force on side PQ

$$\vec{F_1} = I(\overrightarrow{PQ} \times \vec{B}) \qquad or \;\; F_1 = IlB\sin 90^0 = IlB \;.....(1)$$

Again Force on side QR

$$= \vec{F_2} = I(\overrightarrow{QR} \times \vec{B}) \;\; or \qquad F_2 = IlBsin\theta \;\; ....(2)$$

And force on side RS

$$= \vec{F_3} = I(\overrightarrow{RS} \times \vec{B}) \qquad or \;\; F_3 = IlBsin90^0 = IlB \;......(3)$$

Similarly Force on side SP

$$= \vec{F_4} = I(\overrightarrow{SP} \times \vec{B}) \;\; or \;\; F_4 = IlBsin(180-\theta) \Rightarrow F_4 = IlBsin\theta \;.......(4)$$

According to Fleming's left hand rule the Forces $\vec{F_2}$ & $\vec{F_4}$ are equal and act along the axis of the coil in opposite direction so exert no any torque they are cancelled out.

While the force $\vec{F_1}$ & $\vec{F_3}$ will exert a torque on the coil given as

$$\vec{\tau} = Force \times perpendicular\ distance$$

$$or \qquad \tau = IlB \times bsin\theta$$

$$= IBlbsin\theta$$

$$\tau = IBA\ sin\theta \qquad (A = lb)$$

$$0r \qquad \vec{\boldsymbol{\tau}} = (\vec{\boldsymbol{M}} \times \vec{\boldsymbol{B}}) = \boldsymbol{MBsin}\theta$$

Where $M = IA$ = magnetic dipole moment

25 Moving coil Galvanometer:- m.imp

A Galvanometer is a device which is used to detect the current in a circuit.

Principle:-

The Galvanometer is based on the principal that a current carrying coil placed in external magnetic field experience a torque.

Construction:- -

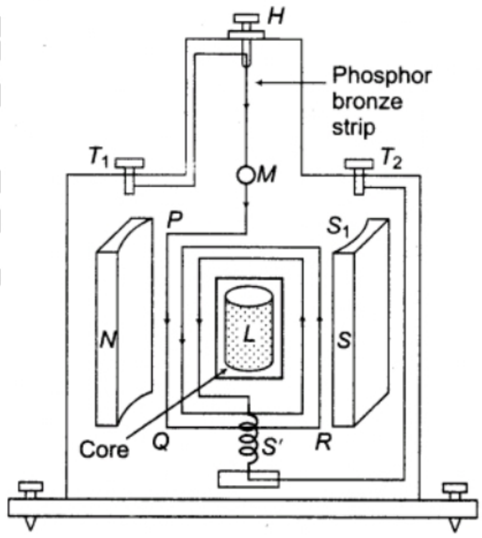
A moving coil Galvanometer consist of

(i) A coil PQRS consist of a large number of turns of fine insulated copper wire wounded over a non magnetic metallic frame. The coil is suspended from a movable torsion head H by mean of a fine phosphor bronze strip. The lower end of coil is connected to fine spiral spring S'. An arrangement of strong north & South Pole is fixed on both sides of the coil.

(ii) A soft iron core L of spherical or cylindrical shape is placed between the coils.

(iii) A circular Mirror M is attached to the Phosphor bronze strip, to measure the deflection of the coil.

(iv) The torsion head is connected to a binding terminal T_1 & spring S' is connected to another binding terminal T_2.



Theory:-

Let N is number turns in coil, A is area of coil, B is magnetic field I is the amount of current following through the coil, then torque acting on the coil is

$$\tau = NIBASin\theta$$

Here θ is angle between normal to the plane of coil & applied magnetic field. Since the field is radial i,e $\theta = 90^0$ *so* $\tau = NIBA$

Due to this torque the phosphor bronze wire suffer twist, if θ is angle of twist & k is moment of restoring couple per unit angular twist then moment of restoring couple = k θ

In equilibrium $NIBA = k\,\theta$

Or $I = \frac{k}{NBA}.\theta$

Here $\frac{k}{NBA}$ = K = a another constant

So $\mathbf{I \propto \theta}$

Hence current through the coil is directly proportional to the deflection of the coil.

Sensitivity of a moving coil Galvanometer

A galvanometer is said to be more sensitive if it show large deflection even for a small change in current is passed through it.

1. Current sensitivity: - it is defined as the deflection produced in the galvanometer when a unit current is passed through it.

   I,e. Current sensitivity $I_s = \frac{\theta}{I} = \frac{NBAI}{kI} = \frac{NBA}{k}$

2. Voltage sensitivity: - It is defined as the deflection produced in the galvanometer when a unit potential difference is applied across it.

   Voltage sensitivity $V_s = \frac{\theta}{V} = \frac{\theta}{IR} = \frac{NBAI}{kIR} = \frac{NBA}{kR}$

   Clearly voltage sensitivity = $\frac{current\ \ sensitivity}{R}$

The sensitivity of a moving coil galvanometer can be increased by

- ✓ By increasing number of turns in the coil.
- ✓ By increasing the magnetic field.
- ✓ By increasing the area of the oil.
- ✓ By decreasing the value of torsion constant R

Advantage of moving coil Galvanometer:-

- It tells about the deflection due to current in a circuit.

Disadvantage of a moving coil Galvanometer:

- Its sensitivity cannot be changed at will.
- It is damaged by overloading.

26 Conversion of a Galvanometer into an ammeter $^{m.imp}$:-

(An ammeter is a device which is used to measure the current in a circuit, as current flow in wire so it is connected in series. An ammeter is formed by connecting a low resistance (shunt) in parallel to Galvanometer. The value of shunt depends upon the current which is to be measured.)

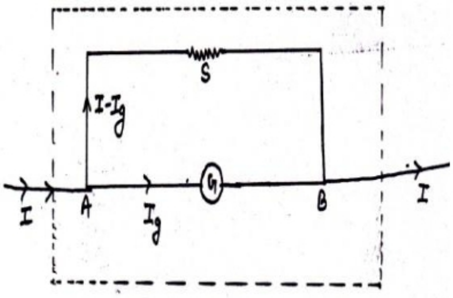
As shunt & Galvanometer are connected in parallel so I_g G = (I – I_g) S

Or $$S = \frac{I_g G}{I - I_g}$$

So by connecting a shunt across galvanometer, we get an ammeter of desired range, moreover

$$S(I - I_g) = I_g G \Rightarrow I_g = \frac{S}{G+S} \times I$$

As the deflection in galvanometer is directly proportional to I_g & hence perpendicular so the current can be measured. Also the effective resistance is R - $\frac{GS}{G+S}$ < S



- *An ideal ammeter has zero resistance.* (imp:-MCQ)
- Higher the range of ammeter to be prepared lower should be the value of shunt.
- The range of ammeter can be increased but cannot be decreased.

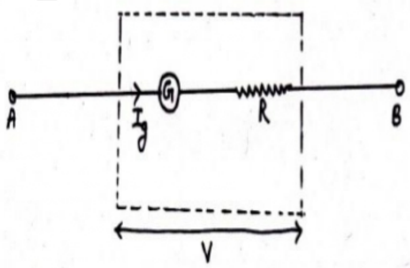
27 Conversion of a galvanometer into a Voltmeter:- m.imp

(Voltmeter is a device which is used to measure the potential difference between two points in a circuit so connected in parallel. A voltmeter is formed by connecting a high resistance in series a Galvanometer.)

The total resistance in the Circuit is $= R + G$

Now by ohm's Law $I_g = \frac{V}{R+G}$ Or $R + G = \frac{V}{I_g}$

$$Or \quad R = \frac{V}{I_g} - G$$



So by connecting a high resistance in Series with G, we can get a voltmeter of desire range. As deflection is proportional to I_g & hence V. So the scale can be arranged to measure potential difference. Hence a voltmeter is a high resistance galvanometer. Its effective resistance is Rv = R+G>>G

- An ideal voltmeter should have infinite resistance. *(imp:-MCQ)*

Example8. In the circuit the current is to be measured. What is the value of the current if the ammeter shown (a) is a galvanometer with a resistance $R_G = 60\ \Omega$; (b) is a galvanometer described in (a) but converted to an ammeter by a shunt resistance rs = 0.02 Ω; (c) is an ideal ammeter with zero resistance?

Solution (a) Total resistance in the circuit is $R_G + 3 = 63\ \Omega$. Hence, $I = \frac{3}{63} = 0.048\ A$

(b) Resistance of the galvanometer converted to an ammeter is, $\frac{R_G R_S}{R_G + R_S} = \frac{60 \times 0.2}{60+0.2} = 0.02\Omega$

Total resistance in the circuit is, $0.02 + 3 = 3.02\ \Omega$ hence, $I = \frac{3}{3.02} = 0.99\ A$

(c) For the ideal ammeter with zero resistance, $I = \frac{3}{3} = 1.00\ A$