Chapter–3: Motion in a Straight Line

In daily life we observe that some substances are in rest & some are in motion. Study of Motion of objects is very important in physics.

1. Mechanics:-

Mechanics is that branch of physics which deals with the motion of objects and the equilibrium of the object under many forces.

Mechanics

- STATICS
- KINEMATICS
- DYNAMICS

- **Statics:-**

It deals with the objects which are at rest or we can say that it deals with the object which is under equilibrium under many force

- **Kinematics:-**

It deals with the motion of object without considering the cause of motion.

- **Dynamics:-**

It deals with the motion of object along with the cause of motion of object.

2. REST AND MOTION:-

- **REST:-**

If an object does not change its position of rest with respect to surrounding than it is called in rest.

E.g.:- a book placed on the table.

- **Motion:-**

If an object changes its position w.r.t to surrounding than it is called in motion.

E g, A moving car on the road etc.

- **Rest & motion are relative terms explain**

Sometimes we see that objects are in rest but actually they are in motion like a person sitting in train see that another persons are in rest but they are in motion.

Similarly sometimes we see that objects are in motion but they found in rest, like a person in vehicle see that trees are moving but actually they are in rest.

Hence we cannot say that a object is in rest or in motion. These are relative term.

3 THE CONCEPT OF A POINT OBJECT:-

When we study motion of an object in physics than the shape & size of the object does not matter. They can be neglected. Also the size of object may also be negligibly small.

I.e. a study of motion of bus on the road, Here size of bus can be taken as a point w.r.t the length of road. Hence in mechanics a particle or a body is a point, of negligible dimensions

4. Motion in one, two & three dimensions:-

- **ONE DIMENSION MOTION:**

A particle moving along a straight line or path is said to be posses' one dimensional motion. I,e only one co-ordinate axis changes w.r.t time. E.g- Motion of train on straight track, motion of bus on straight road etc.

- **TWO DIMENSIONAL MOTIONS:-**

A particle moving in a plane is said to be passes two dimensional motion I.e. only two co-ordinate axis changes w.r.t time. e.g.- Motion of insect on the floor, motion of earth around the sun.

- **THREE DIMENSIONAL MOTIONS:-**

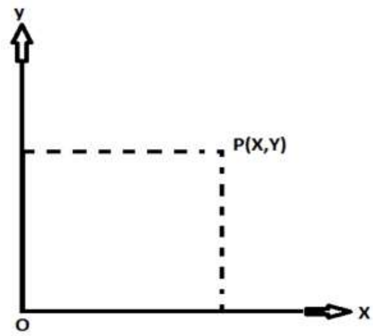
A particle moving in space is said to be passes three dimensional motion I.e. all the three co- ordinate axis changes with respect to time e.g- A bird, an aero plane flying in the sky etc.

5. Frame of reference:-

The fixed point or place with respect to which the position, velocity or acceleration of an object is measured is called frame of reference.

Cartesian co-ordinate system:-

Draw two mutually perpendicular line OX & OY meeting at a point O. Here O is called origin & OX & OY called co-ordinate axis. And the region covered by OX & OY is a plane. This system is called Cartesian co-ordinate system. Similarly by drawing three mutually perpendicular axes we can study space.



Cartesian co-ordinate system are of two types:-

(1)- Inertial frame of reference which is either at rest or moving with uniform velocity is called inertial frame of reference. It obeys Newton's law of motion.

(2)- Non inertial frame of reference:-Any accelerated frame of reference is called non inertial frame of reference. It does not obey Newton's law of motion. Earth is a non inertial frame of reference.

6. TYPES OF MOTION:-

A body can have three type of motion:-

(i)- Translational motion:-

If line joining the two points remains parallel to itself throughout the motion is called translational motion. e.g. A bus moving on straight road.

(ii)- Rational motion:-

If a body moving around a fixed axis is called rotational motion. E.g. Motion of earth around sun etc.

(iii)- Vibration motion:-

If a body moves to & fro about a mean position after a regular interval of time is called Vibration motion. E.g.- Motion of pendulum.

7. DISTANCE AND DISPLACEMENT:-

DISTANCE:-

The length or actual path followed by an object between two points in a given interval of time is called distance.

DISPLACEMENT:-

The straight line path between any two objects is called displacement.

***Give any five differences between distance and displacement**

Distance	Displacement
1 Distance is actual path followed by an object between two points.	1 Displacement is the straight line distance followed by the object.
2 It is a scalar quantity	2 It is a vector quantity.
3 It is always +ve	3 It may be +ve, -ve or zero.
4 Distance may be equal to or Greater than displacement	4 Displacement may be equal to or smaller than distance
5 Distance per unit time is speed.	5 Displacement per unit time is velocity.

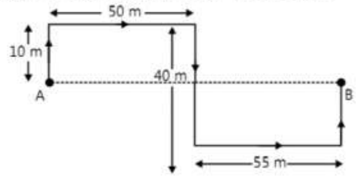
QUESTION 1: A person moves on a semi-circular track of radius 40.0 m during a morning walk. If he starts at one end of the track and reaches at the other end, find the distance covered and the displacement of the person.

Solution : The distance covered by the person equals the length of the track.

It is equal to $\pi R = \pi \times 40.0\, m = 126\, m$.

The displacement is equal to the diameter of the semi-circular track joining the two ends. It is $2R = 2 \times 40 = 80m$

QUESTION 2: Find the distance and displacement of a particle travelling from one point to another, say from pt. A to B, in a given path.



Sol: Total distance travelled = 10+50+40+55+ (40 - 10) = 185 m

Total Displacement = 50 + 55 =105 m

8. SPEED AND VELOCITY:-

(a) Speed:-

The distance travelled by the body per unit time is called speed.

$$\text{Speed}=\frac{\text{distance travelled}}{time\ taken}$$

Unit of speed:- m/s in **S.I,** in c,g,s system cm/sec.

*speed is a scalar quantity. It may be +ve or zero

Types of speed

1. **Uniform Speed:-**

   A particle or body is said to be moving with uniform speed if it cover equal distance in equal interval of time. Sometime it is called constant speed.

2. **Variable velocity:**

Particle or body is said to be moving with variable speed if it cover unequal distance in equal interval of time or equal distance in unequal interval of time.

3. **Average speed:-**

Average speed of a body is defined as the ratio of total distance travelled by the body to the total time taken.

$$V_{av} = \frac{\text{total distance travelled}}{Total\ time\ taken}$$

*It may be noted that particle may or may not have speed equal to average speed at any instant.

*Speed is equal to average speed only when particle moves with uniform speed.

QUESTION 3: A man walks at a speed of 6 km/hr for 1 km and 8 km/hr for the next 1 km. What is his average speed for the walk of 2 km ?

Solution : Distance travelled is 2 km. Time taken $\frac{1\ km}{6\ km/hr} + \frac{1\ km}{8km/hr} = \left(\frac{1}{6}+\frac{1}{8}\right) = \frac{7}{24}hr$

Average speed $= \frac{2\ km \times 24}{7\ hr} = \frac{48}{7} km/hr = 7\ km/hr.$

4. **Instantaneous speed:-**

The speed of an object at any instant of time is called instantaneous speed To calculate Instantaneous we have to make Δx as small as possible so that the speed of the body does not change during interval. Hence $V_{ins} = \text{Lim} \frac{\Delta x}{\Delta t}$

(B) Velocity:-

The displacement of a particle or body per unit time is called velocity of the body. .

$$v = \frac{\text{displacement}}{\text{time}}$$

Velocity is a vector quantity. It can be +ve, -ve & zero.

Types of velocity:-

(1)- Uniform velocity (ii) variable velocity (iii) Average velocity (iv) Instantaneous Velocity.

* All these velocities can be defined by types of speed by only replacing speed by velocity & distance by displacement.

Difference between speed & velocity:-

Speed	**Velocity**
It is the ratio of distance to the time.	It is the ratio of displacement to the time.
It is a scalar quantity.	It is a vector quantity.
It may be +ve or zero.	It may be +ve,-ve or zero
It has only magnitude.	It has both magnitude & direction.
Speed may be equal to or greater than velocity.	Velocity may be equal to or smaller than speed

QUESTION 4: If a train moves from station A to B with a constant speed v =40 km/h and returns back to the initial point A with a constant speed V2=30 km/h, then calculate the average speed and average velocity.

Sol: Average speed is distance covered divided by time taken. Distance is length of the path travelled. Average velocity is displacement divided by time taken. Displacement is the vector from initial point to final point.

Let the distance AB = s, Time taken by train from A to B, $t_1 = \frac{s}{v_1}$

Time taken by train FromB to A, $t_2 = \frac{s}{v_2}$ $Average\ speed = \frac{Total\ distance}{Total\ time\ taken} = \frac{s+s}{t_1+t_2} = \frac{2s}{\frac{s}{v_1}+\frac{s}{v_2}}$

$v_{av} = \frac{2s}{\frac{s}{v_1}+\frac{s}{v_2}} = \frac{2v_1v_2}{v_1+v_2} = \frac{2\times40\times30}{40+30} = 34.3 km/h$ $Average\ velocity = \frac{Net\ displ\ acement}{Total\ time} = \frac{0}{t_1+t_2} = 0$

QUESTION 5: The distance travelled by a particle in time t is given by $s = (2.5)t^2$.

Find (a) the average speed of the particle during the time 0 to 5.0 s, (b) the instantaneous speed at t = 5.0 s.

Solution : (a) The distance travelled during time 0 to 5.0 s is $s = (2.5)(5.0)^2 = 62.5\ m.$

The average speed during this time is $v. = \frac{62.5m}{5\ s} = 12.5\ m/s.$

(b) $s = (2.5)t^2$ $or,$ $\frac{ds}{dt} = (2.5)(2t) = (5.0)\ t.$

At t = 5.0 s the speed is $v = \frac{ds}{dt} = (5.0)(5.0) = 25\ m/s.$

QUESTION 6: A table clock has its minute hand 4.0 cm long. Find the average velocity of the tip of the minute hand (a) between 6.00 a.m. To 6.30 a.m. And (b) between 6.00 a.m. to 6.30 p.m.

Solution : At 6.00 a.m. the tip of the minute hand is at 12 mark and at 6.30 a.m. or 6.30 p.m. it is 180° away. Thus, the straight line distance between the initial and final position of the tip is equal to the diameter of the clock. So $Displacement = 2R = 2 \times 4.0\ cm = 8.0\ cm.$

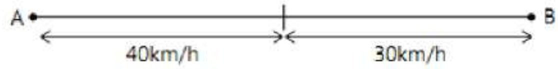
(a) The time taken from 6.00 a.m. to 6.30 a.m. is 30 minutes = 1800 s.

The average velocity is $V_{av} = \frac{displacement}{time} = \frac{8.0}{1800} = 4.4 \times 10^{-3}$cm/s

(b) The time taken from 6.00 a.m. to 6.30 p.m. is 12 hours and 30 minutes = 45000 s.

The $average\ velocity = \frac{Displacement}{time} = \frac{8.0\ \text{cm}}{45000\ \text{s}} = 1.8 \times 10^{-4}$ cm/s

QUESTION 7: Consider a train moving from station A to B with a constant speed of 40 km/h for half the time and with constant speed of 30 km/h for the next half time of that journey. Calculate the average speed of the whole journey.



Sol: Let AB = s and T = Total time of journey.

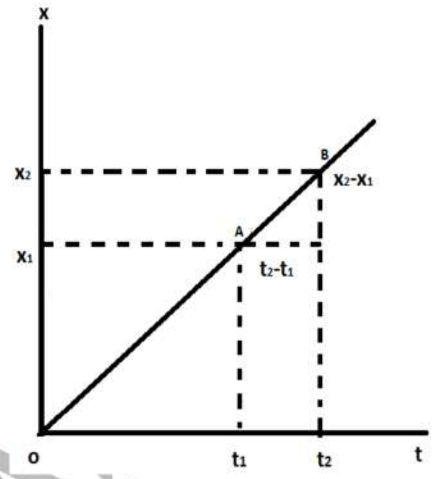
Distance travelled in first half time $\frac{T}{2}$ is, $s_1 = v_1 \frac{T}{2}$.

Distance travelled in second half time is, $s_2 = \frac{v_2 T}{2}$

Or $Average\ speed = \frac{Total\ distance}{Total\ time}$ or $v_{av} = \frac{v_1(T/2) + v_2(T/2)}{T} = 35\ km/h$

9. DISPLACEMENT TIME GRAPH

For uniform motion displacement time graph or position time graph is a straight line making an angle O with X axis.



Determination of velocity from position-time graph.

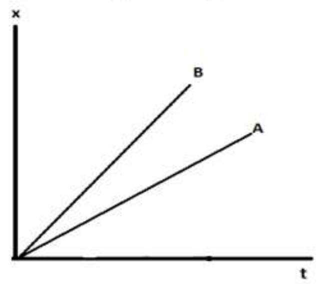
Consider two points A & B on the straight line the position & time corresponding to A & B are (T_1, X_1) & (T_2,x_2) respectively.

Now the displacement of the body co-responding to time interval (T_2-T_1) is (X_2-X_1)So, Velocity of the body= $\frac{\text{displacement}}{\text{time}}$

OR $v = \frac{X_2 - X_1}{T_2 - T_1} = \frac{BC}{AC} = \tan\theta$

$\frac{BC}{AC} = \tan\Theta$ Represents the slop of the line.

Hence we can say that velocity of a body having uniform motion = Slope of the displacement time graph



QUESTION 8: Two bodies having their displacement time graph as shown in fig. which one have larger velocity.

Ans:- Slope of B is greater so B have larger velocity.

Question9: A woman starts from her home at 9.00 a.m., walks with a speed of 5 kmh^{-1} on a straight road up to her office 2.5 km away, stays at the office up to 5.00 p.m. and returns home by an auto with a speed of $25\ kmh^{-1}$. Choose suitable scales and plot the x-t graph of the motion

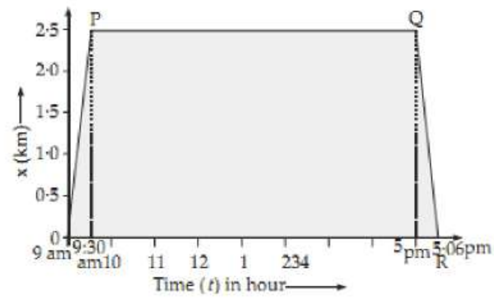
Answer Let $v_1 = 5\ kmh^{-1}$ $x = 2.5\ km$

If t_1 = time taken to reach office, then it can be calculated by using the formula $x = v_1 t_1 \implies t_1 = \frac{x}{v_1}$

Hence , $t_1 = \frac{2.5}{5} = \frac{1}{2}h = 30\ minutes.$

When she stays at her office from 9.30 a.m. to 5.00 p.m., she is at rest and her stay is represented by the straight line PQ in the graph. On return, speed of auto, $v_2 = 25\ km/h$. Let, t_2 = time taken by her, i.e. by auto from office to her home, then $t_2 = \frac{x}{v_1} = \frac{2.5}{25} = \frac{1}{10}$ hour = 6 minutes. So, she reaches back to her home at 5.06 p.m. Her motion on the return journey is shown by QR part of the graph.

Scale taken Time on x-axis, 1 division = 1 hour. Distance on y-axis, 1 division = 0.5 km.



10. VELOCITY-TIME GRAPH FOR UNIFORM MOTION:-

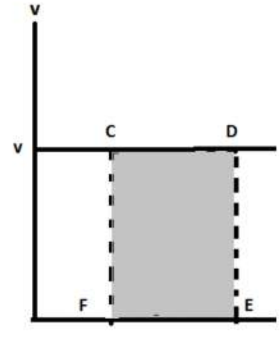
Velocity time graph for uniform motion is a straight line parallel to time axis. The area under the velocity time graph gives the displacement of the particle. Consider any two points C & D on the line corresponding to time T_1&T_2.Then the area of CDEF can be given as

Area of CDEF= CD x CF $= (t_2 - t_1)\ V$ –(i)

As we know V = $\frac{X_2 - X_1}{t_2 - t_1}$

So from eqn (i) Area of CDEF = $\frac{X_2 - X_1}{t_2 - t_1}(t_2 - t_1) = X_2 - X_1$ = displacement

Hence area under velocity time graph = displacement of the body.



11. Relative velocity:-

Relative velocity of a body B w.r.t is the rate at which b changes its position relative to A.

Let us consider three cases in this

(a)If both the particles are moving in the same direction then $V_{rAB} = V_A - V_b$ (in magnitude)
(b)If the particles or bodies are moving in opposite direction then $V_{rAB} = V_A + V_b$

(c) If the two particles are moving in the mutually perpendicular directions than $V_{rAB} = \sqrt{V_a{}^2 + V_B^2}$

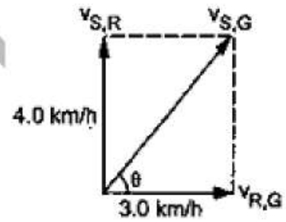
(d)If the angle between V_a & V_b is θ than $V_{rAB} = \sqrt{V_A^2 + V_B^2 - 2V_aV_B \cos\theta}$

QUESTION 10: A swimmer can swim in rest water at a rate 4.0 km/h. If he swims in a river flowing at 3.0 km/h and keeps his direction perpendicular to water, find his velocity with respect to the ground.

Solution :The velocity of the swimmer with respect to water is $v_s = 4.0\,km/h$ in the direction perpendicular to the river. Figure shows the velocities.

It is clear that, $V_{s,G} = \sqrt{(4.0)^2 + (3.0)^2} = 5.0\,km/h$.

The angle θ made with the direction of flow is $tan\theta = \frac{4}{3}$



12. Acceleration:-

The rate of change of velocity is defined as the acceleration of the body

$$\text{I,e } a = \frac{dv}{dt}$$

- If the speed is increasing then it is called +ve acceleration & if the speed is decreasing than it is called –ve acceleration.
- It is a vector quantity.
- c,g,s unit – cm/sec^2, S.I unit - m/s^2, dimention formula [M^o L^1 T^{-2}]
- Retardation: If speed of a body decreases then acceleration of the body is called retardation
  - Negative acceleration does not imply retardation.
  - Retardation refers to decrease in speed and not velocity.

*** Types:-**

Uniform acceleration, non uniform acceleration, average acceleration, instantaneous acceleration.

13. DIFFERENT TYPES OF ACCELERATION:-

(i)- Uniform acceleration:-

The acceleration of an object is said to be uniform if its velocity changes by equal amount in equal interval of time. However small these interval may be.

(ii)- Variable acceleration:-

The acceleration of ab object is said to be non uniform or variable if its velocity changes by unequal amount in equal interval of time.

(iii)- <u>Average Acceleration</u>:-

Average Acceleration is defined as the ratio of total change in velocity of the object to the total time taken.

$$a_{av} = \frac{V_2 - V_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

(iv)- <u>Instantaneous Acceleration</u>:-

The acceleration of an object at a given instant of time is called instantaneous acceleration.

$$a = \lim \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

As $$V = \frac{dx}{dt}$$

$$\Rightarrow \quad a = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

* Thus acceleration is the first order derivative of velocity & second order derivative of displacement with respect to time.

QUESTION 11: For a particle moving along x-axis, displacement time equation is $x = 20 + t^3 - 12t$.

(a) Find the position and velocity of the particle at time $t = 0$

(b) Find out whether the motion is uniformly accelerated or not.

(c) Find out the position of particle when velocity is zero.

Sol: (a) as $x = 20 + t^3 - 12t \quad \ldots(i)$

At $t = 0, \quad x = 20 + 0 - 0 = 20\,m$

By differentiating Equation (i) w.r.t. time i.e. $v = \frac{dx}{dt} = 3t^2 - 12$ ... (ii)

At $\quad t = 0, v = 0 - 12 = -12\,m/s$

(b) Differentiating equation (ii) w.r.t. time, we get the acceleration $a = \frac{dv}{dt} = 6t$

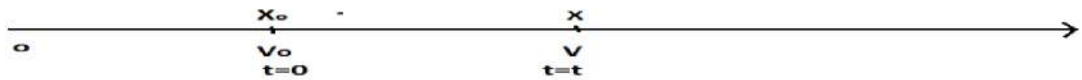
As acceleration is a function of time, the motion is non-uniformly accelerated.

(c) Substituting $v = 0$ in equation (ii) $0 = 3t^2 - 12 \quad or \quad t = 2$ sec.

Substituting it in equation (i) we have $x = 20 + (2)3 - 12(2)$ or $x = 4\,m$

<u>14. Kinematic equation for uniformly accelerated motion along straight line:-</u>

Let us consider an object moving with constant acceleration along +ve direction of X axis as shown in fig.



Let $\quad x_o$= Position of object at time t = 0 $\quad$ X = Position of object at time t = t.

U or V_o= Velocity of object at time t=0 $\quad$ V= Velocity of object at time t=t.

(a)- VELOCITY – TIME RELATIONSHIP-

As we know Acceleration = $\frac{\text{change in velocity}}{\text{Time taken}}$

or $a=\frac{v-v_o}{t-0}$

Or $a=\frac{v-v_o}{t}$

Or $at= v-v_o$ Or $v= u+at$

(b)- POSITION TIME RELATIONSHIP:-

As average velocity of an object in the time interval o to t can be given as :

$$v_{av} =\frac{\text{Displacement}}{\text{time}} = \frac{x-x_o}{t-0}$$

Or $x-x_o = v_{av} . t$ -----(i)

Also $v_{av}=\frac{\text{initial velocity} + \text{final velocity}}{2}$

or $v_{av} = \frac{v_o+v}{2}$ ----………………..(ii)

Putting in eq^n (i) we have $x-x_o = (\frac{v_o+v}{2})t$

or , $x-x_o =(\frac{2v_o+at}{2})t$ ($\because v= v_o+at$)

$$x-x_o=v_ot+\frac{1}{2}at^2$$

here $x-x_o$= distance travelled in t times = S

So, $S= ut + \frac{1}{2}at^2$

POSITION VELOCITY RELATIONSHIP:-

We know that $v= v_o+ at$ or $v- v_o = at$ ---(i)

Also average velocity × time = displacement

Or $(\frac{v+v_o}{2})t = x- x_o$

$= v+ v_o =\frac{2}{t}(x- x_o)$ ---(2)

Multiply eqn (1) & (2) $(v- v_o)(v+ v_o)= at \times \frac{2}{t}(x- x_o)$

$$v^2 - v^2o = 2a(x- x_o)$$

Or $v^2 - u^2 = 2as$

QUESTION 12 : A car moving along a straight highway with speed of 126 kmh^{-1} is brought to a stop within a distance of 200 m. What is the retardation of the car (assumed uniform) and how long does it take for the car to stop ?

Ans. Initial velocity of car, $u = 126\, kmh^{-1} = 126 \times \frac{5}{18}\ ms^{-1} = 35\, ms^{-1}$ ...(i)

Since, the car finally comes to rest, $v = 0$ Distance covered, $s = 200\, m, a = ?, t = ?$

From equation of motion $v^2 = u^2 - 2as$ or, $a = \frac{v^2-u^2}{2s}$ ……… …(ii)

Substituting the values from eq. (i) in eq.(ii) $a = -\ 3.06ms^{-2}$

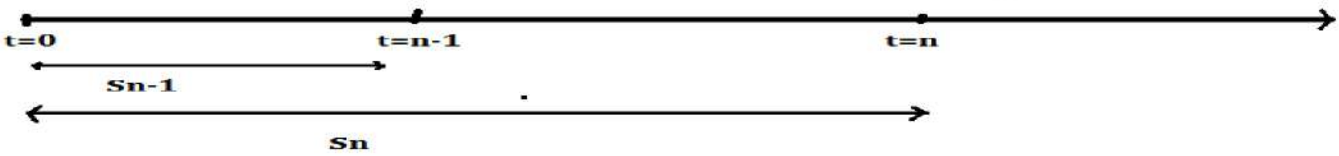
Negative sign shows that car is uniformly retarded at $-\ a = 3.06\ ms^{-2}$.

To find t, let us use the relation $v = u + at \quad or \quad t = \frac{v-u}{a}$

Here, a = $-\ 3.06\ ms^{-2}, v = 0, u = 35\ ms^{-1} \ so \ t = 11.44\ sec.$

DISTANCE COVERED IN N th SECOND :-

Distance covered in nth interval can be obtained by subtracting the distance travelled in first (n-1) second from the distance travelled in first n second.



The distance travelled in first t second is given by $s_t = v_o\ t + \frac{1}{2}\ at^2$

The distance travelled in first (n-1) second can be given as $s_{n-1} = v_{o(n-1)} + \frac{1}{2}\ a(n-1)^2$

The distance travelled in first n second is $s_n = v_o n + \frac{1}{2}\ an^2$

Hence distance travelled in nth second is

$$s_{nth} = s_n - s_{n-1} = v_o\, n + \frac{1}{2}an^2 - [v_{o(n-1)} + \frac{1}{2}\ a(n-1)^2]$$

$$= v_o\, n + \frac{1}{2}an^2 - v_{o(n-1)} - \frac{1}{2}\ a(n-1)^2$$

$$= v_o\, n + \frac{1}{2}an^2 - v_o n + v_o - \frac{1}{2}\ a(n^2 + 1 - 2n)$$

$$= \frac{1}{2}an^2 + v_o - \frac{1}{2}\ an^2 - \frac{1}{2}\ a + an$$

$$S_{nth} = v_o + \frac{a}{2}\ (2n-1)$$

* This equation is often used to calculate the displacement in the "nth second". However, as you can verify, different terms in this equation have different dimensions and hence the above equation is dimensionally incorrect.
* Correct form is $S_{nth} = v_o(1\ s) + \frac{a}{2}\ (2n - 1(1s))\ (1s)$
* Also note that this equation gives the displacement of the particle in the last 1 second and not necessarily the distance covered in that second.

QUESTION 13: A particle having initial velocity u moves with a constant acceleration a for a time t. (a) Find the displacement of the particle in the last 1 second. (b) Evaluate it for u = 5 m/s, a = 2 in/s 2 and t =10 s.

Solution : (a) The position at time t is S= $ut + \frac{1}{2}at^2$

The position at time (t -1 s) is $s' = u(t-1) + \frac{1}{2}\ a(t-1)^2 = ut - u + \frac{1}{2}\ at^2 - at + \frac{1}{2}a$

Thus, the displacement in the last 1 s is $s_t = s - s' = u + at - \frac{1}{2}a \quad or, \quad s_t = u + \frac{a}{2}(2\,t - 1)$

(b) Putting the given values in (i) = $5 + \frac{1}{2} \times 2\ (2 \times 10 - 1) = 5\,m + 19\,m = 24\,m.$

QUESTION 14: A body moving with uniform acceleration covers 24 m in the 4th second and 36 m in the 6th second. Calculate the acceleration and initial velocity.

Sol: We know the formula for displacement in nth second. For 4th second and 6th second we get two equations and two variables u and a. So we solve the equations to get the values of u and a.

$S_{nth} = v_o + \frac{a}{2}(2n-1) \Rightarrow$ for $t = 4\,sec$ we have $24 = u + \frac{a}{2}(2 \times 4 - 1)$ ……… 1

and for $t = 6\,sec$ we have $36 = u + \frac{a}{2}(2 \times 6 - 1)$ ……..2

solving both equations we get $12 = 2a \Rightarrow a = 6\,m/s^2$ so from equation(1) $u = 3m/s$

15. EQUATION OF MOTION BY CALCULUS METHOD(Integration &differentiation method)

First equation of motion:-

Acceleration is defined as $a = \frac{dv}{dt}$

Or $dv = adt$ --(i)

When time = 0 velocity = u (say) and when time= t velocity = v (say)

Integrating eq^n (i) we get $\int_u^u dv = a\int_o^t dt$

$= [v]^v_u = a[t]^t_o$

or $v - u = at$

Or $v = u + at$

(b) Second equation of motion:-

Velocity is defined as $v = \frac{ds}{dt}$

Or $ds = vdt$ ---(**2**)

When time = 0 than velocity = u (say) When time t = t than velocity = v (say)

Integrating eq^n(2) we get

$$\int_o^s ds = \int_o^t (u + at)dt = \int_o^t udt + \int_o^t atdt$$

$$S = ut + a[\frac{t^2}{2}]_o^t$$

$$S = ut + \frac{1}{2}at^2$$

(C) Third equation of motion :-

By the definition of acceleration & velocity $a = \frac{dv}{dt} = \frac{dv}{ds} \times \frac{ds}{dt} = \frac{dv}{ds} \times v$

Or $ads = vdv$ ---(3)

When time = 0, velocity =u distance travelled = 0 and

When time = t, velocity =v, distance travelled = s

Integrating eq^n ---(**3**) we get $\int_o^s ads = \int_u^v vdv$

$$a\,s = [\frac{v^2}{2}]_u^v$$

$$a\,s = \frac{v^2}{2} - \frac{u^2}{2} = \frac{1}{2}(v^2 - u^2)$$

or $v^2 - u^2 = 2as$

Fourth equation of motion:-

By definition of velocity $v = \frac{ds}{dt}$

$$ds = vdt = (u+at)dt = udt + atdt$$

Or $ds = udt + atdt$ ---(**4**)

When time = (n-1) second, distance travelled = s_{n-1}

When time = n second, distance travelled = s_n

Integrating eq^n (4) we get, $\int_{sn-1}^{sn} ds = u \int_{n-1}^{n} dt + a \int_{n-1}^{n} t dt$

$$S_n - S_{n-1} = u[t]^n_{n-1} + a\left[\frac{t^2}{2}\right]^n_{n-1}$$

$$S_{nth} = u\,[n-(n-1)] + a\left[\frac{n^2}{2} - \frac{(n-1)^2}{2}\right]$$

$$S_{nth} = un - un + u + \frac{an^2}{2} - \frac{a}{2}(n^2 + 1 - 2n)$$

$$S_{nth} = u + \frac{an^2}{2} - \frac{an^2}{2} - \frac{a}{2} + an$$

$$S_{nth} = u + \frac{a}{2}(2n-1)$$

16. MOTION UNDER GRAVITY:-

When body is thrown upwards or body is falling freely under gravity than acceleration of the body changes into acceleration due to gravity.

Equation of motion for a freely falling body:-

For a freely falling body, the following equation of motion hold good.

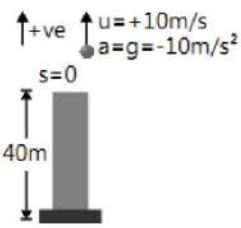
(i) $v = u + gt$ (ii) $s = ut + \frac{1}{2}gt^2$ (iii) $v^2 - u^2 = 2gs$ (iv) $S_{nth} = u + \frac{g}{2}(2n-1)$

When a body is thrown vertically upwards then eq^n of motion becomes :-

(i) $v = u - gt$ (ii) $S = ut - \frac{1}{2}gt^2$ (iii) $v^2 - u^2 = -2gs$ (iv) $S_{nth} = u - \frac{g}{2}(2n-1)$

When body is falling freely than acceleration is taken +ve & when body is thrown vertically upwards then acceleration is taken negative.

QUESTION 15: A ball is thrown upwards from 40 m high tower with a velocity of 10 m/s. Calculate the time when it strikes the ground. ($g = 10\ m/s^2$)



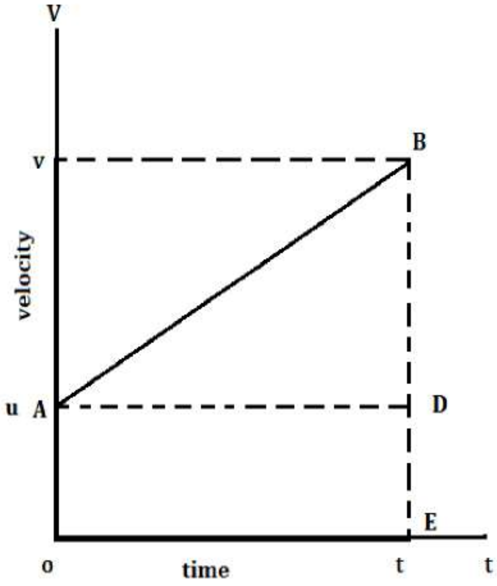
Sol: $u = +10\ m/s,\ a = -10\ m/s^2\ s = -40\ m$ (at the point where the ball strikes the ground) As $s = ut + \frac{1}{2}gt^2 \implies -40 = 10t - 5t^2$

$\implies t = 4\ s, -2\ s.\ Considering\ the\ positive\ value, t = 4\ s$

17. DERIVATION OF EQUATION OF MOTION BY GRAPHICAL METHOD:-

Equation of motion by graphical method:-

Consider an object moving along a straight line with initial velocity u and uniform acceleration a. suppose it travels s distance in t time. Here,



OA = ED =u, OC = EB = v, OE = AD = t

(a)First eq^n of motion:-

We know that Acceleration = slop of velocity time graph AB

Or $a = \frac{DB}{AD} = \frac{DB}{OE} = \frac{EB-ED}{OE} = \frac{v-v}{t}$

Or v=u+at

(b)Second eq^n of motion:-

As $a= \frac{DB}{AD} = \frac{DB}{t} \rightarrow$ DB =at

Distance travelled by the object in t time is

S = Area under the trapezium OABE

= Area of rectangle OADE + area of triangle ADB

$= OA\times OE + \frac{1}{2} DB \times AD$

$= ut + \frac{1}{2} at \times t$

Or $s= ut + \frac{1}{2} at^2$

(C)Third equation of motion:-

Distance travelled by object in time t is

S = area of trapezium OABE

$S=\frac{1}{2}(EB + OA) \times OE = \frac{1}{2}(EB + ED) \times OE$

Also acceleration a= slope of velocity time graph AB

Or $a = \frac{DB}{AD} = \frac{EB-ED}{OE}$ or $OE = \frac{EB-ED}{a}$

So $s = \frac{1}{2}(EB + ED)\frac{EB-ED}{a}$

Or $2as = (EB)^2 - (ED)^2$

$2as = v^2 - u^2$

Or $v^2 - u^2 = 2as$