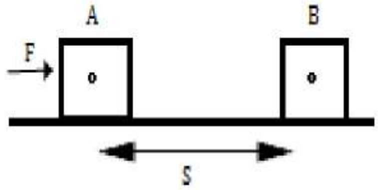
**Chapter–6: Work, Energy and Power**

## 1. WORK

Work is said to be done when a force applied on a body displace the body through a distance in the direction of force.




Let us consider that force F displace the body from point A to point B through a distance S. than work done to displace the body is $\boldsymbol{W = \vec{F}.\vec{S}}$

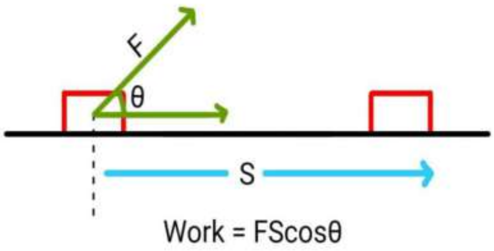
**When force act at an angle θ with horizontal:-**

Suppose a force F is applied on a body at an angle θ with the horizontal and displace the body from point A to B through a distance S. Now resolving F into components

i. $F\ Cos\theta$ in direction of displacement.
ii. $F\ Sin\theta$ in perpendicular direction of displacement.

Clearly here the motion is due to $Fcos\theta$ so $W = FS\ cos\theta$

or $\boldsymbol{W = \vec{F}.\vec{S}}$




Thus work done by a force is equal to dot product of force and displacement of the body.

**Dimensions and Unit of Work:-**

As work=force × displacement So [W] =[MLT$^{-2}$][L] **Or** **[W]= [ML$^2$T$^{-2}$]**

**Unit of work** There are of two types of unit of work

**1. Absolute unit of work:-**

(i) **In SI the absolute unit of work is Joule.**

If $F = 1N\ and\ S = 1\ metre$ than $1\ Joule = 1N \times 1m$

Thus, work done is said to be one joule if one Newton force displaces the body through one meter in its own direction.

(ii) **In c,g,s system the absolute unit of work is erg.**

If $F = 1\ dyne\ and\ S = 1cm\ then\ 1\ Erg = 1\ dyne \times 1\ cm.$

Hence work done is said to be one erg if one dyne force displace a body to one metre distance in its own direction. The relation between one joule and one erg is

$$\mathbf{1\ J = 10^7\ erg}$$

## 2. Gravitational unit of work:-

(i) In SI the gravitational unit of work is kilogram metre

$$1\,kilogram\,metre = 1kg\,wt \times 1m = 9.8N \times 1m$$

or $$\mathbf{1kg\,m = 9.8\ J}$$

Thus, work done is said to be one kg m if one kg wt force displaces the body through one meter distance in its own direction.

(ii) In cgs the gravitational unit of work is Gram centimeter

$$1\,Gram\,centimeter = 1g\,wt \times 1\,cm = 980\,dyne \times 1\,cm.$$

**1g cm=980 erg**

Thus, work done is said to be one g cm if one g wt force displaces the body through one centimeter distance in its own direction.

**Question 1 A body of mass 2 kg initially at rest moves under the action of an applied horizontal force of 7 N on a table with coefficient of kinetic friction = 0.1. Compute the**
**(a) Work done by the applied force in 10 s**
**(b) Work done by friction in 10 s**
**(c) Work done by the net force on the body in 10 s**
**(d) Change in kinetic energy of the body in 10 s and interpret your results.**

**Answer:** (a) We know that $frictional\,force = \mu_k \times normal\,reaction$

Or $= 0.1 \times 2\,kg\,wt = 0.1 \times 2 \times 9.8\,N = 1.96\,N$

or $net\,effective\,force = (7 - 1.96)\,N = 5.04\,N$

so $acceleration = \frac{F}{m} = \frac{5.04}{2}\,ms^{-2} = 2.52\,ms^{-2}$

and distance traveled , $S = \frac{1}{2}at^2 = \frac{1}{2} \times 2.52 \times 10 \times 10 = 126\,m$

so work done by applied force $= FS = 7 \times 126\,J = 882\,J$

(b) Work done by friction $= fS = 1.96 \times 126 = -246.96\,J$

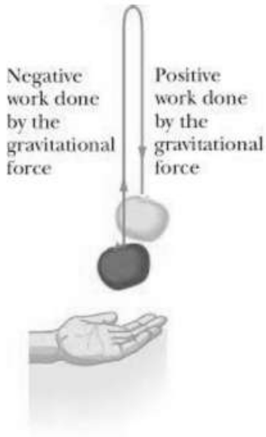
(c) Work done by net force $= 5.04 \times 126 = 635.04\,J$

(d) Change in the kinetic energy of the body = work done by the net force in 10 seconds = 635.04 J

**Question 2. A trolley of mass 300 kg carrying a sandbag of 25 kg is moving uniformly with a speed of 27 km/h on a friction less track. After a while, sand starts leaking out of a hole on the trolley's floor at the rate of 0.05 kg $s^{-1}$. What is the speed of the trolley after the entire sand bag is empty?**

**Answer:** The system of trolley and sandbag is moving with a uniform speed. Clearly, the system is not being acted upon by an external force. If the sand leaks out, even then no external force acts. So there shall be no change in the speed of the trolley.

## 2. Nature of work done:-




**1 If $\theta < 90^0$ than $Cos\theta$ is $+ve$ so $W = FS\ (+ve)$**

(a) When a body is lifted, the work done by the lifting force is positive.

(b) When a spring is stretched, work done by the stretching force is positive.

**2 If $\theta = 90^0$ than $= FScos90^0 = 0$**

(a) A person holding a heavy stone at rest is doing no work. I.e. S=0

(b) A coolie carrying a suitcase on his head is doing no work. I.e. $\theta=90^0$

**3 If $\theta > 90^0$ than $W = -FS\ (-ve)$**

(a)When break are applied on a moving vehicle, the work done by the breaking force is negative.

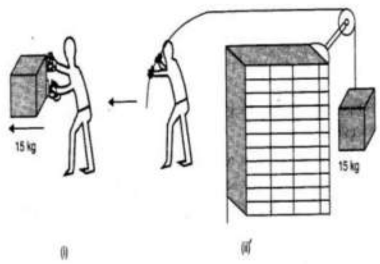
(b) Work done by the frictional force is negative.

**Question 3. The sign of work done by a force on a body is important to understand. State carefully if the following quantities are positive or negative:**
**(a) Work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket,**
**(b) Work done by gravitational force in the above case,**
**(c) Work done by friction on a body sliding down an inclined plane,**
**(d) Work done by an applied force on a body moving on a rough horizontal plane with uniform velocity,**
**(e) Work done by the resistive force of air on a vibrating pendulum in bringing it to rest.**

**Answer:** Work done, $W = F.S = FS\cos\theta$

(a) Work done 'positive', because force is acting in the direction of displacement i.e., θ = 0°.
(b) Work done is negative, because force is acting against the displacement i.e., θ = 180°.
(c) Work done is negative, because force of friction is acting against the displacement i.e., θ=180°.
(d) Work done is positive, because body moves in the direction of applied force i.e., θ= 0°.
(e) Work done is negative, because the resistive force of air opposes the motion i.e., θ = 180°.

**Question 4 : In Fig.(i), the man walks 2 m carrying a mass of 15 kg on his hands. In Fig. (ii), he walks the same distance pulling the rope behind him. The rope goes over a pulley, and a mass of 15 kg hangs at its other end. In which case is the work done greater?**




**Answer:** In Fig. (i), force is applied on the mass, by the man in vertically upward direction but distance is moved along the horizontal. $\theta = 90°$. $W = Fs\cos 90° = zero$

In Fig. (ii), force is applied along the horizontal and the distance moved is also along the horizontal. Therefore, θ = 0°.

$$W = Fs\cos\theta = mg \times s\cos 0°$$
$$W = 15 \times 9.8 \times 2 \times 1 = 294\ joule.$$

Thus, work done in (ii) case is greater.

## 3. Work Done In Term of Rectangular Components:-

In term of rectangular components the force and displacement can be given as

$$\vec{F} = F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k} \quad and \quad \vec{S} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$

∴ Work $\boldsymbol{W} = \vec{\boldsymbol{F}}.\vec{\boldsymbol{S}} = (F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k}).(x\hat{\imath} + y\hat{\jmath} + z\hat{k}) = xF_x + yF_y + zF_z$

**Question 5. A body constrained to move along the z-axis of a coordinate system is subject to a constant force F given by $\vec{F} = -\hat{\imath} + 2\hat{\jmath} + 3\hat{k}N$ where i, j, k, are unit vectors along the x- y- and z-axis of the system respectively. What is the work done by this force in moving the body a distance of 4 m along the z-axis?**
**Answer: since the body is displaced 4m along Z-axis only,**

**So displacement vector** $\vec{S} = -0\hat{\imath} + 0\hat{\jmath} + 4\hat{k}$ Also $\vec{F} = -\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

So work done $W = \vec{F}.\vec{S} = (-\hat{\imath} + 2\hat{\jmath} + 3\hat{k}).(-0\hat{\imath} + 0\hat{\jmath} + 4\hat{k}) = 12\hat{k}.\hat{k} = 12\ joule$

## 4. CONSERVATIVE AND NON CONSERVATIVE FORCES:- M.Imp

**(a) Conservative force:-** A force is said to be conservative if the amount of work done by or against the force does not depend on the path followed by the body but depend only on the initial and final position of the body. E.g. gravitational force, electromagnetic force .

**(b) Non-conservative force:-** A force is said to be non-conservative if the amount of work done by or against the force depend on the path followed by the body. E.g. frictional force.

### Show that Gravitational Force is conservative in nature:-

Suppose a body of mass M is raised to a height h by the following four steps. If equal work is done to move the body to same height than work done by the gravitational force is conservative in nature.

(i)As shown in fig. (1) as the body is raised vertically upward to a height h by the force $F = mg$

So work done $=W = \vec{F}.\vec{S} = FScos\theta$

∴ $W_1 = mghcos0^0 = mgh$ (1)

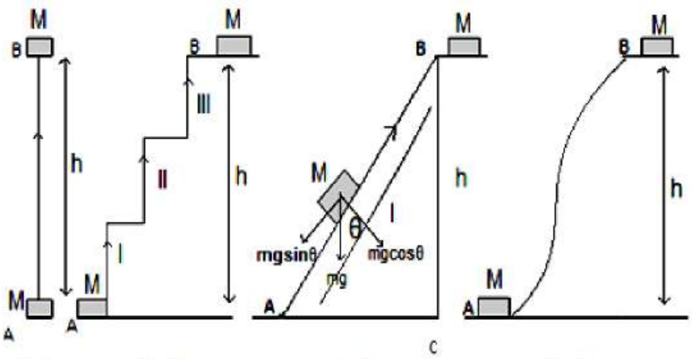


fig-1 fig-2 fig-3 fig-4

(ii)Again consider Fig (2) here body moves upward stepwise through steps I, II &III at a height $BC = h$. Here when body moves horizontal then work done is zero and the total work done is equal to the total vertical distance covered by the body , hence

$$W = mg(h_1 + h_2 + h_3) = mgh \qquad (2)$$

(iii) Now as shown in fig (3).here body moves at an inclined plane inclined at an angle θ

Here clearly $F = mgSin\theta\ along\ AB$

Than $W_2 = FS = mg \times \frac{BC}{AB} \times AB = mg \times h$

$\Longrightarrow$ $W_2 = \text{mgh}$

(iv) Now as shown in fig (4).here body moves in zig- zag path .this path can be assumed to form from many infinite small horizontal and vertical segments .here also total work done is equal to vertical segments so $W = mgh$

Hence we can say that work done by the gravitational force on a body is independent of path, so it is conservative in nature.

**Question 6. A rain drop of radius 2 mm falls from a height of 500 m above the ground. It falls with decreasing acceleration (due to viscous resistance of the air) until at half its original height, it attains its maximum (terminal) speed, and moves with uniform speed thereafter. What is the work done by the gravitational force on the drop in the first and second half of its journey? What is the work done by the resistive force in the entire journey if its speed on reaching the ground is 10 $ms^{-1}$ ?**

**Answer:** Here,.r = 2 mm = $2 \times 10^{-3}$ m

Distance moved in each half of the journey,.S $= \frac{500}{2} =$ 250 m

Density of water, ρ = $10^3$ kg/m$^3$ So Mass of rain drop = volume of drop × density

$$m = \frac{4}{3}\pi r^2 \rho = \frac{4}{3} \times \frac{22}{7}(2 \times 10^{-3}) \times 3 \times 10^3 = 3.35 \times 10^{-5}\ \text{kg}$$

$$\text{So } W = mgS = 3.35 \times 10^{-5} \times 9.8 \times 250 = 0.082\ \text{J}$$

Note: Whether the drop moves with decreasing acceleration or with uniform speed, work done by the gravitational force on the drop remains the same. If there was no resistive forces, energy of drop on reaching the ground.

$$E_1 = mgh = 3.35 \times 10^{-5} \times 9.8 \times 500 = 0.164\ \text{J}$$

Actual energy,$E_2 = \frac{1}{2}mv^2 = \frac{1}{2} \times 3.35 \times 10^{-5}(10)^2 = 1.675 \times 10^{-3}$J

Work done by the resistive forces,$W = E_1 - E_2 = 0.164 - 1.675 \times 10^{-3} W = 0.1623\ J$

## 5. ENERGY

Energy is defined as ability of a body to do work. Energy can be measured by the total amount of work that a body can do.

Unit of energy is same as that of work i.e. joule

**TYPES OF ENERGY:-**

Energy of is of many type i.e. solar energy, wind energy, electrical energy, mechanical energy

In this chapter we will discuss only mechanical energy.

## 6. MECHENICAL ENERGY

Mechanical energy is defined as sum of potential energy and kinetic energy

### (a)KINETIC ENERGY

The energy possess by a body due to its motion is called kinetic energy.

e.g. flowing water, moving vehicle possess kinetic energy

**Expression for kinetic energy :-** let us consider a body is in rest initially, now a force F is applied on the body due to which it moves a distance S with v velocity then

As $v^2-u^2=2aS$

So $v^2=2aS \quad (\because u=0)$

So $S=\frac{v^2}{2a}$ (1)

Also F=ma (2)

From equation (1) and (2) as $W=FS=ma\times\frac{v^2}{2a}=\frac{1}{2}mv^2$

And this work done is stored as kinetic energy of the body. So kinetic energy $=\frac{1}{2}mv^2$

**Relation between K.E. and linear momentum:-**

As we know the kinetic energy of the body is $K.E=\frac{1}{2}mv^2$

Now dividing and multiplying the above equation by m in RHS we get

$$KE=\frac{1}{2}\frac{m^2v^2}{m}=\frac{1}{2}\frac{P^2}{m} \qquad (\because\ p=mv)$$

So we obtain that K.E $\propto$ square of linear momentum

Special cases

(i) If P =constant, K.E $\propto \frac{1}{m}$

(ii) If K.E =constant, $P^2 \propto \sqrt{m}$

(iii) If m=constant, $P\propto\sqrt{K.E}$ as shown in fig respectively.

**Question 7. An electron and a proton are detected in a cosmic ray experiment, the first with kinetic energy 10 keV, and the second with 100 keV. Which is faster, the electron or the proton? Obtain the ratio of their speeds, (electron mass = 9.11 x $10^{-31}$ kg, proton mass = 1.67 x $10^{-27}$ kg, 1 eV= 1.60 x $10^{19}$J).**

**Answer:** $K_e = 10\ keV\ \ and\ K_p = 100keV\ also\ m_e = 9.11 \times 10^{-31}kg\ and\ m_P = 1.67 \times 10^{-27}kg$

As we know $K = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2K}{m}}$ or $\frac{v_e}{v_p} = \sqrt{\frac{K_e \times m_p}{K_p \times m_e}} = \sqrt{\frac{10\times1.67\times10^{-27}}{100\times9.11\times10^{-31}}} = 13.54$

$$v_e = 13.54\ v_p$$

**Thus electron is travelling faster.**

## (b)POTENTIAL ENERGY

The energy possess by a body due to its position is called potential energy

E.g. the water stored in a dam possesses potential energy which can be converted in to kinetic energy.A stretched spring possess potential energy.

**(i) Gravitational potential energy**⟹ it is due to position above the surface of earth.

**(ii) Elastic potential energy** ⟹ it is due to compression and stretching of body(I,e-spring)

**(i)Expression for gravitational potential energy:-** Let us consider a body of mass m is raised to a height h, by applying a force F=mg in upward direction, then work done by the force is

$$W = F.h = F\ hcos\theta = Fhcos0^0$$

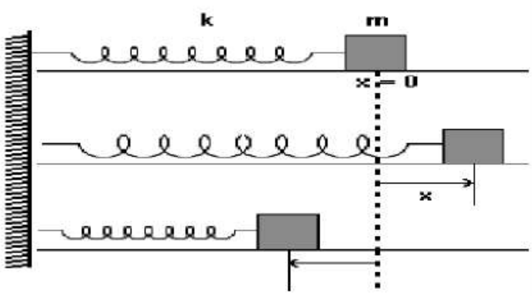
$$Or \qquad W = Fh\ or\ W = mgh$$

☺ Work done by the gravitational energy is $W = mg.hcos180^0 = -mgh.$

**Potential energy is of two types:-**

**(ii)POTENTIAL ENERGY OF A SPRING (elastic potential energy)**

When a spring is stretched or compressed from its normal position (x=0) by a small distance x, then a restoring force is produced in the spring to bring it to normal position. The restoring force so produced is proportional to the displacement x in opposite direction i.e




$$\vec{F} \propto -\vec{x}\ \ or\ \vec{F} = -k\,\vec{x}$$

Where k is called spring constant and the above relation is called Hooks Law.

Now work done to move the small distance $dx$ is $dw = -kx.dx$

total work done to move the spring x against restoring force is distance is $w = \int kx.dx$

Or $$w = -k\int x.dx = -\frac{kx^2}{2}$$

This work is stored in the body in form of potential energy. So the potential energy of the spring is

$$U = \frac{1}{2}kx^2$$

☺External force is just equal and opposite of restoring force i,e $\vec{F}_{ext} = -\vec{F}$

hence potential energy of a spring is defined as the energy possessed due to its compression and expansion.

**Spring constant** :-

Spring constant is defined as restoring force per unit displacement of the spring $k = \frac{F}{x}$

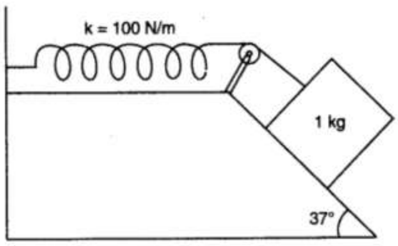
**Question 8. A bolt of mass 0.3 kg falls from the ceiling of an elevator moving down with a uniform speed of 7 $ms^{-1}$. It hits the floor of the elevator (length of elevator = 3 m) and does not rebound. What is the heat produced by the impact? Would your answer be different if the elevator were stationary ?**

**Answer:** P.E. of bolt = mgh = 0.3 ×9.8 × 3 = 8.82 J

The bolt does not rebound. So the whole of the energy is converted into heat.

Since the value of acceleration due to gravity is the same in all inertial system, therefore the answer will not change even if the elevator is stationary.

**Question 9. A 1 kg block situated on a rough incline is connected to a spring with spring constant 100 $Nm^{-1}$ as shown in Figure. The block is released from rest with the spring in the unstretched position. The block moves 10 cm down the incline before coming to rest. Find the coefficient of friction between the block and the incline. Assume that the spring has negligible mass and the pulley is friction less.**




**Answer: from the above figure $R = mgcos\theta$ and $F = \mu R = \mu mgcos\theta$**

**Net force on the block down the incline $= mgsin\theta - F$**

$$= mgsin\theta - \mu mgcos\theta = mg(sin\theta - \mu cos\theta)$$

**Here distance moved $x = 10cm = 0.1m$**

**In equilibrium work done = potential energy of stretched spring**

$$= mg(sin\theta - \mu cos\theta)x = \frac{1}{2}kx^2$$

$$= 2\,mg(sin\theta - \mu cos\theta) = kx$$

**Or** $$= 2 \times 1 \times 10(0.601 - \mu \times 0.798) = 10$$

**Or** $= (0.601 - \mu \times 0.798) = \frac{10}{20} = 0.5 \quad \Rightarrow \mu = 0.126$

## 7. CONGERVATIVE NATURE OF ENERGY M.Imp

**Law of conservation of energy:-**Energy can neither be created nor be destroyed but it can be converted from one form to another form.

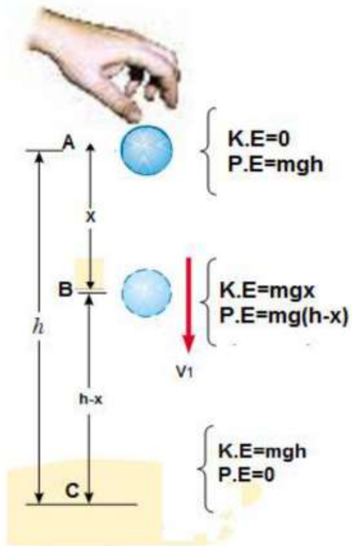
**Proof conservation of energy of a freely falling body:-**

The mechanical energy (K.E.+P.E.) is constant in case of freely falling body**.** Let us consider the body is initially at rest at a height h from ground

then at point A Kinetic Energy $= \frac{1}{2}mv^2 = 0$ And Potential Energy = mgh

So the mechanical energy at $A = P.E + K.E = mgh + o = mgh$……………….(1)

Now let body fall freely from A to B with velocity $v_1$such that A B = x. So the height of B from the ground is (h-x)




As we know that $v_1^2 - u^2 = 2aS \text{ here } u = 0, a = g \text{ and } s = x$

So $K.E = \frac{1}{2}mv_1^2 = \frac{m}{2} \times 2gx = mgx$

Similarly P.E. = mg (h- $x$)

Hence the mechanical energy at point B = mg (h- $x$ ) + mg $x$

Mechanical Energy = mgh- mg $x$ + mg $x$ = mgh………………….(2)

Now let the body just reaches the ground point C so h=0

Here potential Energy =mg× (0) =0

Let $v_2$ is the velocity of the body at just reaching the point C

Using $v^2 - u^2 = 2aS$ we have $v_2^2 = 2gh$

So $K.E = \frac{1}{2}mv_2^2 = \frac{1}{2}m \times 2gh = mgh$

Hence mechanical energy at point C =K.E+P.E=mgh+0=mgh…….. (3)

From equation (1), (2) and (3), it is clear that total mechanical energy of a body during the free fall of a body under gravity remain constant. When body reaches the ground the kinetic energy is converted into heat energy and sound energy

**Question 10 : An artificial satellite orbiting around earth loses its energy gradually due to atmospheric resistance. Why then does its speed increase progressively as it comes closer and closer to the earth?**

Answer: As an artificial satellite gradually loses its energy due to dissipation against atmospheric resistance, its potential decreases rapidly. As a result, kinetic energy of satellite slightly increases i.e., its speed increases progressively.

**Question 11. A person trying to lose weight (dieter) lifts a 10 kg mass, one thousand times, to a height of 0.5 m each time. Assume that the potential energy lost each time she lowers the mass is dissipated, (a) How much work does she do against the gravitational force? (b) Fat supplies 3.8 x $10^7$J of energy per kilogram which is converted to mechanical energy with a 20% efficiency rate. How much fat will the dieter use up?**

**Answer:** Here, $m = 10\,kg,\ h = 0.5\,m,\ n = 1000$

(a) work done against gravitational force. $W = n(mgh) = 1000 \times (10 \times 9.8 \times 0.5) = 49000J.$

(b) Mechanical energy supplied by 1 kg of fat = $3.8 \times 10^7 \times \frac{20}{100} = 0.76 \times 10^7\ J/kg$

.-. Fat used up by the dieter =$\frac{1kg}{0.76 \times 10^7} \times 49000 = 6.45 \times 10^{-3}\ kg$

**Question 12. A rain drop of radius 2 mm falls from a height of 500 m above the ground. It falls with decreasing acceleration (due to viscous resistance of the air) until at half its original height, it attains its maximum (terminal) speed, and moves with uniform speed thereafter. What is the work done by the gravitational force on the drop in the first and second half of its journey? What is the work done by the resistive force in the entire journey if its speed on reaching the ground is 10 $ms^{-1}$ ?**

**Answer:** Here,.r $= 2\text{ mm} = 2 \times 10^{-3}$ m

Distance moved in each half of the journey,.S $= \frac{500}{2} = 250$ m

Density of water, $\rho = 10^3\ kg/m^3$ So Mass of rain drop = volume of drop × density

$$m = \frac{4}{3}\pi r^2 \rho = \frac{4}{3} \times \frac{22}{7}(2 \times 10^{-3}) \times 3 \times 10^3 = 3.35 \times 10^{-5}\ kg$$

$$\text{So } W = mgS = 3.35 \times 10^{-5} \times 9.8 \times 250 = 0.082\ J$$

Note: Whether the drop moves with decreasing acceleration or with uniform speed, work done by the gravitational force on the drop remains the same. If there was no resistive forces, energy of drop on reaching the ground.

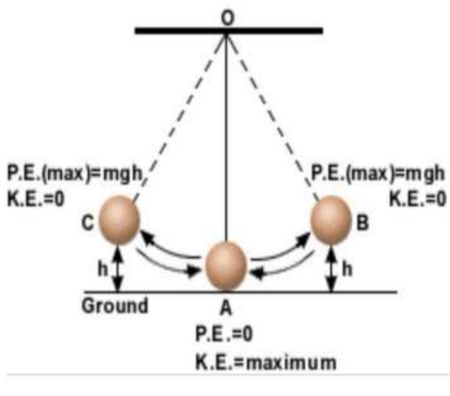
$$E_1 = mgh = 3.35 \times 10^{-5} \times 9.8 \times 500 = 0.164\ J$$

Actual energy,$E_2 = \frac{1}{2}mv^2 = \frac{1}{2} \times 3.35 \times 10^{-5}(10)^2 = 1.675 \times 10^{-3}J$

Work done by the resistive forces,$W = E_1 - E_2 = 0.164 - 1.675 \times 10^{-3} W = 0.1623J$

## 8. VIBRATION OF A SIMPLE PENDULUM:-

Let us consider initially bob of pendulum is released from B. at point B initially the P.E. is maximum now on releasing from B its potential energy decrease and K.E increases.




At point A its K.E becomes maximum. Now when bob moves upward towards points C again its potential energy increases and Kinetic energy decreases now at point C again its potential energy becomes maximum. Hence during the complete cycle of pendulum there is conservation of energy.

**Question 13. The bob of a pendulum is released from a horizontal position. If the length of the pendulum is 1.5 m, what is the speed with which the bob arrives at the lowermost point, given that it dissipated 5% of its initial energy against air resistance?**

**Answer:** On releasing the bob of pendulum from horizontal position, it falls vertically downward by a distance equal to length of pendulum i.e., $h = l = 1.5\,m$ .As 5% of loss in P.E. is dissipated against air resistance, the balance 95% energy is transformed into K.E. Hence,

$$\frac{1}{2}mv^2 = \frac{95}{100}\times mgh \Rightarrow v = \sqrt{2\times\frac{95}{100}\times gh} = \sqrt{2\times 95\times 9.8\times\frac{1.5}{100}} = 5.3\text{ms}^{-1}$$

## 9. Different forms of energy:-

- **Heat energy**:-The energy possessed by the molecules due to their random motion is called heat energy.
- **Internal energy: -** The energy possessed by the body due to temperature and intermolecular forces is called internal energy
- **Electrical energy: -** the energy possessed by the body due to motion of charge particles called electrical energy.
- **Nuclear energy: -** the energy required to hold the nucleons inside a nucleus is called nuclear energy.

## 10. Transformation of Energy:-

The phenomenon of change of one form of energy into another form is called transformation of energy. For example

- In a dam the potential energy of the water due to its height is converted into kinetic energy then into electrical energy with the help of a turbine.
- In an electric bulb, the electric energy is converted into light energy and heat energy.
- In a thermal power plant the chemical energy stored in coal is converted into electric energy with the help of a turbine.

Hence one form of energy can be converted into another form.

## 11. VARIATION OF MASS WITH VELOCITY:-

According to Einstein, the variation of mass of a body with velocity is given by $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$

Where $m_o$ is the rest mass of the body v is the velocity of the body and $c=3\times10^8$ $ms^{-1}$ is the velocity of light. Let the body have velocity v equal to velocity of light than $m = \frac{m_0}{\sqrt{1-\frac{c^2}{c^2}}} = \frac{m_0}{\sqrt{1-1}} = \frac{m_0}{0} = \infty$

I,e A body having $\infty$ mass means acceleration produced in the body =0 I,e $a = \frac{F}{\infty} = 0$

Thus no material particle can achieve velocity greater than velocity of light.

## 12. EINSTEIN'S MASS-ENERGY RELATIONSHIP:-

According to Einstein's mass and energy are related to each other as $E=mc^2$

Hence mass of a body can be converted into energy and energy can be converted into mass.

## 13. WORK ENERGY THEOREM M.Imp

We know that small amount of work done to move a body through small distance $dx$ may be given as $$dW = F.dx$$

Now to calculate total work done to move body from point A to B may be given as

$$W_{AB} = \int_{v_1}^{v_2} F.dx = \int_{v_1}^{v_2} ma.dx = \int_{v_1}^{v_2} m\frac{dv}{dt}.dx \text{..........(1)}$$

As we know $v = \frac{dx}{dt} \Longrightarrow dx = vdt$

So using in (1) $W_{AB} = \int_{v_1}^{v_2} m\frac{dv}{dt}.vdt = \int_{v_1}^{v_2} mvdv = m\int_{v_1}^{v_2} vdv$

$$W_{AB} = m\int_{v_1}^{v_2} vdv = m\left[\frac{v^2}{2}\right]_{v_1}^{v_2}$$

$$W_{AB} = \frac{m{v_2}^2}{2} - \frac{m{v_1}^2}{2} = K_B - K_A$$

Or W =Final Velocity –Initial Velocity

Or W= change in kinetic energy =Δ K.E

Hence work done is equal to change in kinetic energy of a body this relationship is called work energy theorem.

## 14. POWER M.Imp

Power is defined as the rate at which work is done. It is a scalar quantity i,e

$$P = \frac{dW}{dt} \quad or \quad P = \frac{d(\vec{F}.\vec{S})}{dt} = \vec{F}.\frac{\overrightarrow{dS}}{dt} = \vec{F}.\vec{v}$$

Hence power may also be defined as the dot product of the force and the velocity.

Dimensional formula of power can be given as $P = \frac{W}{t} = \frac{M^1L^2T^{-2}}{T^1} = [M^1L^2T^{-3}]$

**Unit of power:-** The S.I. unit of power is Watt

$$1Watt = \frac{1\,joule}{1Sec}$$

Hence power is said to be one Watt if one Joule work is done in one second.

Power is also measured in

- 1 kilowatt =1000 watt
- 1 megawatt=$10^6$ watt
- 1 horse power =746 watt (this unit is used in engineering)
- The cgs unit of power is erg/sec as 1watt =$10^7$ erg/sec

**Question 14. A body is initially at rest. It undergoes one-dimensional motion with constant acceleration. The power delivered to it at time t is proportional to (i) $t^{1/2}$ (ii) t (iii) $t^{3/2}$ (iv) $t^2$**

**Answer:** (ii) From $v = u + at$ we get $v = 0 + at = at$

As power, $P = F \times v$ or $p = (ma) \times at = ma^2t$

Since m and a are constants, therefore, P α t.

**Question 15. A body is moving unidirectional under the influence of a source of constant power. Its displacement in time t is proportional to (i) $t^{1/2}$ (ii) t (iii) $t^{3/2}$ (iv) $t^2$**

**Answer:** (ii) $P = force\ x\ velocity$

Dimensional formula of power $[P] = [F][v] = [MLT^{-2}][LT^{-1}] = [ML^2T^{-3}]$

As power is constant so $L^2T^{-3} = constant \Rightarrow \frac{L^2}{T^3} = constant$

Or $L^2 \alpha T^3 \Rightarrow L\ \alpha\ T^{3/2}$

**Question 16. A pump on the ground floor of a building can pump up water to fill a tank of volume 30 $m^3$in 15 min. If the tank is 40 m above the ground, and the efficiency of the pump is 30%, how much electric power is consumed by the pump?**

**Answer:** Here, $volume\ of\ water = 30\ m^3$ ; $t = 15\ min = 15 \times 60 = 900\ s,\ h = 40\ m\ ;\ n = 30\%$

As the density of water = $\rho = 10^3\ kg\ m^{-3}$

Mass of water pumped, $m = volume \times density = 30 \times 10^3\ kg$

Actual power consumed or output power $P_0 = \frac{W}{t} = \frac{mgh}{t}$ => $P_0 = \frac{30 \times 103 \times 9.8 \times 40}{900} = 13070\ watt$

If $P_i$ is input power (required), then as $\eta = \frac{P_0}{P_i}$ => $P_i = \frac{P_0}{\eta} = \frac{13070}{\frac{30}{100}} = 43567\ W = 43.56\ KW$

## 15. COLLISION

The term collision refers to the interaction between to body or two particles due to which the direction of the magnitude of the colliding particles changes

### Types of collision

**(a)Perfectly elastic collision:-** A collision between two particles is said to be perfectly elastic if both linear momentum and velocity of the system remain conserved.

e.g. collision between atomic particles and collision between two glass ball are nearly perfectly elastic collision

**(b)Inelastic collision:-**

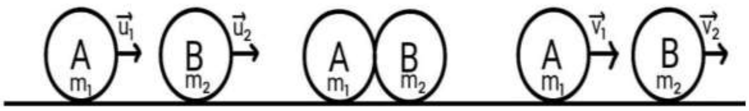
A collision is said to be inelastic if the linear momentum of the system remain conserved but its kinetic energy is not conserved

**Perfectly inelastic collision:-**A collision is said to be perfectly inelastic collision if the two bodies after collision stick together and move as one body. In this case linear momentum remain conserved

E.g. when a bullet hit the wooden block and move with the block is a example of nearly perfectly inelastic collision

## 16. ELASTIC COLLISION IN ONE DIMENTIONTAL MOTION M.Imp

Let us consider two bodies of mass $m_1$ and $m_2$ collide with each other having initial velocity $u_1$ and $u_2$ and final velocity $v_1$ and $v_2$ respectively.




Then total linear momentum before collision $= m_1u_1+m_2u_2$

And total momentum after collision $= m_1v_1+m_2v_2$

Now according to law of conservation of linear momentum

Or $$m_1u_1 + m_2u_2 = m_1v_1 = m_2v_2$$

Or $$m_1(u_1 - v_1) = m_2(v_2 - u_2) \qquad (1)$$

Also as collision is elastic in nature so kinetic energy is also conserved in nature i.e

$$\frac{1}{2}m_1u_1^2+\frac{1}{2}m_2u_2^2=\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2$$

or $$m_1\left(u_1^2-v_1^2\right)=m_2\left(v_2^2-u_2^2\right)$$

or $$m_1(u_1-v_1)(u_1+v_1)=m_2(v_2-u_2)(v_2+u_2) \qquad (2)$$

Now dividing eqns. (2) by (1). We get

$$\frac{m_1(u_1-v_1)(u_1+v_1)}{m_1(u_1-v_1)}=\frac{m_2(v_2-u_2)(v_2+u_2)}{m_2(v_2-u_2)}$$

Or $$(u_1+v_1)=(v_2+u_2)$$

Or $$u_1-u_2=v_2-v_1 \qquad (3)$$

i.e. inelastic one dimension motion relative velocity of approach is equal to relative velocity of separation.

**Now velocity after collision:-**

From equation (3) $v_2 = u_1 - u_2 + v_1$ (4)

Substituting value of $v_2$ in equation (1) we get

$$m_1 (u_1-v_1) = m_2 (u_1-u_2+v_1-u_2)$$

$$= m_1 (u_1-v_1) = m_2 (u_1-2u_2+v_1)$$

$$= m_1u_1 - m_1v_1 = m_2 (u_1-2u_2+v_1)$$

$$= m_1u_1 - m_1v_1 = m_2u_1-2m_2u_2+ m_2v_1$$

$$= v_1(m_1+ m_2)= m_1u_1+2m_2u_2-m_2u_1$$

$$\Rightarrow v_1 = \frac{m_1u_1 + 2m_2u_2 - m_2u_1}{m_1 + m_2}$$

Or $$v_1 = \frac{2m_2u_2 - u_1(m_1 - m_2)}{m_1 + m_2} \quad \text{....................(5)}$$

Similarly $$v_2 = \frac{2m_1u_1 - u_2(m_2 - m_1)}{m_1 + m_2} \quad \text{......................(6)}$$

Special case

(1) when the body have equal mass I,e $m_1 = m_2$ (say)

Than from (5) $v_1 = \frac{2mu_2}{2m} = u_2$

Also from (6) $v_2 = \frac{2mv_1}{2m} = v_1$

Hence in one dimensional elastic collision two bodies having equal masses changes their velocities after collision.

(ii) When body B is initially at rest I,e $u_2 = 0$ Then eqns. (5) and (6) becomes

$$v_1 = \frac{u_1(m_1 - m_2)}{m_1 + m_2} \quad \text{....................(7)}$$

And $$v_2 = \frac{2m_1u_1}{m_1 + m_2} \quad \text{......................(8)}$$

Here three sub cases arise as given below.

(i) If both the bodies have same masses ($m_1=m_2$) then from eqns. (7)&(8) we have $v_1=0$ & $v_2=u_1$. Hence in one dimensional elastic collision if the bodies have equal masses then after collision the interchange there velocities.

(ii) If body B is lighter than body A I,e $m_2< m_1$ then $m_2$ can be neglected eqns. (7) and (8) becomes

$$v_1 = \frac{m_1}{m_2}u_1 \quad \& \quad v_2 = \frac{2m_1}{m_1}u_1 = 2u_1$$

Hence if a heavy body collides with a lighter body then there is no change in the velocity of heavy body but the velocity of lighter body becomes twice the velocity of heavy body.

(iii) When body A is lighter then body B I,e $m_1<<m_2$. Then eqns. (7)& (8) becomes

$$v_1 = \frac{-m_2}{m_2}u_1 = -u_1 \quad \text{And} \quad v_2 = \frac{2m_1u_1}{m_2} = 0$$

Hence when a lighter body collides with a heavy body at rest then there is no change in heavy body and the speed of lighter body reverses.

**Coefficient of Restitution:-** M.Imp

It is defined as the ratio of velocity of separation after collision to the velocity of approach before collision; it is represented by e .as

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

The smallest value of e can be zero and the largest value of e can be 1. For perfectly elastic collision the value of e=1.

(i) e=1 means velocity of separation is equal to velocity of approach, means there is no lose in the kinetic energy hence the collision is perfectly elastic collision.

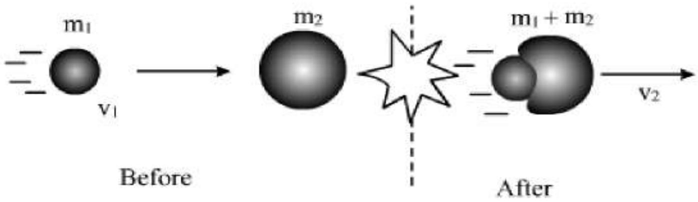
(ii) e=0 means velocity of separation is equal to zero, means the kinetic energy of the colliding bodies is changed into the other form of energy as heat energy, sound energy etc. hence the collision is perfectly inelastic collision.

## 17. PERFACTLY INELASTIC COLLISION IN ONE SIMENTION

*When the two colliding bodies stick together and moves as a single body after collision then the collision is called perfectly inelastic collision.* Let us consider two bodies A and B of mass $m_1$ and $m_2$.now again suppose that body a moving with velocity $u_1$ collide to body B at rest. After collision both the bodies moves with the common velocities v. Now according to conservation of linear momentum

$$m_1 u_1 + m_2 \times 0 = (m_1 + m_2)v$$

or $$v = \frac{m_1}{m_1 + m_2} u_1$$




and the lose in kinetic energy on the collision is

$$\Delta K = K_i - K_f = \frac{1}{2} m_1 u_1^2 - \frac{1}{2}(m_1 + m_2)v^2$$

$$= \frac{1}{2} m_1 u_1^2 - \frac{1}{2}(m_1 + m_2)\left[\frac{m_1}{m_1 + m_2} u_1\right]^2$$

$$= \frac{1}{2} m_1 u_1^2 - \frac{1}{2}\frac{m_1^2}{m_1 + m_2} u_1^2 = \frac{1}{2} m_1 u_1^2\left[1 - \frac{m_1}{m_1 + m_2}\right]$$

Or $$\Delta K = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2} u_1^2$$

This is a positive quantity thus there is a lose in kinetic energy in inelastic collision, this lose is in form of heat energy and sound energy, the total energy in inelastic collision however remains conserved.

**Question 17. A bullet of mass 0.012 kg and horizontal speed 70 ms$^{-1}$ strikes a block of wood of mass 0.4 kg and instantly comes to rest with respect to the block. The block is suspended from the ceiling by thin wire. Calculate the height to which the block rises.**

**Answer:** Here, $m_1 = 0.012\,kg,\ u_1 = 70\,m/s \quad m_2 = 0.4\,kg, \quad u_2 = 0$

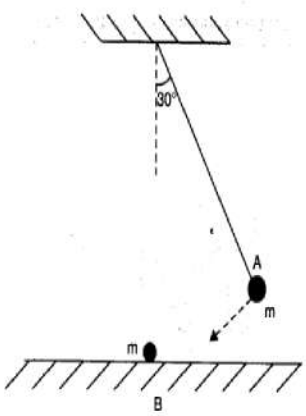
As the bullet comes to rest with respect to the block, the two behave as one body. Let v be the velocity acquired by the combination. Applying principle of conservation of linear momentum,

$$(m_1 + m_2)\,v = m_1 u_1 + m_2 u_2 = m_1 u_1$$

Or $$v = \frac{m_1 u_1}{m_1 + m_2} = \frac{0.012 \times 70}{0.012 + 0.4} = \frac{0.84}{0.412} = 2.04\,ms^{-1}$$

Let the block is raised to height h. As potential energy of the combination= kinetic energy of the combination

So $(m_1 + m_2)\,gh = \frac{1}{2}(m_1 + m_2)\,v^2$ or $h = \frac{v^2}{2g} = \frac{2.04 \times 2.04}{2 \times 9.8} = 0.212\,m$



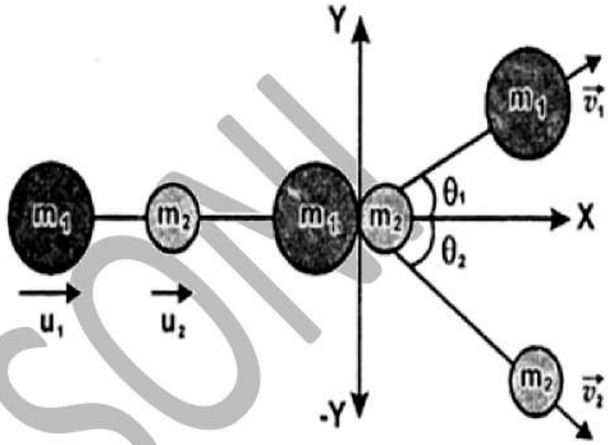

**Question 18. The bob A of a pendulum released from 30° to the vertical hits another bob B of the same mass at rest on a table as shown in Fig. How high does the bob A rise after the collision? Neglect the size of the bobs and assume the collision to be elastic.**

**Answer:** Since collision is elastic therefore A would come to rest and B would begin to move with the velocity of A. The bob transfers its entire momentum to the ball on the table. The bob does not rise at all.

## 18. ELASTIC COLLISION IN TWO DIMENTION OR OBLIQUE COLLISION

Let us consider two bodies A and B of mass $m_1$ and $m_2$.now again suppose that body a moving with velocity $u_1$ collide to body B at rest. After collision suppose the bodies moves with the velocities $v_1$ and $v_2$ making angle $\theta_1$ & $\theta_2$ with the x axis .

Now resolving the linear momentum into rectangular components of body A and B we get




Linear momentum of body A is

$m_1v_1\cos\theta_1$ along x axis

and $m_1v_1\sin\theta_1$ along y axis

similarly linear momentum of body B is

$m_2v_2\cos\theta_2$ along x axis

and $m_2v_2\sin\theta_2$ along y axis

now Appling the principle of conservation of linear momentum along x axis we get

$$m_1u_1 = m_1v_1\cos\theta_1 + m_2v_2\cos\theta_2 \quad\text{......................(1)}$$

As there is no any motion of the bodies along Y axis before collision so the linear momentum along y axis is

$$0 = m_1v_1\sin\theta_1 + m_2v_2\sin\theta_2 \quad\text{......................(2)}$$

Also the kinetic energy also remains conserved in the elastic collision so we may write

$$\frac{1}{2}m_1u_1^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad\text{......................(3)}$$

Here we can see that there are three equations and four variables $v_1, v_2, \theta_1, \theta_2$. Four variable can not be calculated with the help of four equations so any one variable may be calculated experimentally by the help of which three other may be calculated.

Special cases

(i) Glancing collision:- a collision in which the incident particle does not lose any kinetic energy and scatter almost undeflected called glancing collision.

For such collision $\theta_1 = 0^0$ and $\theta_2 = 90^0$

From equation (1)&(2) we get $u_1=v_1$ and $v_2=0$

So the kinetic energy of target particle =0

(ii) head on collision:-the collision in which the target particle moves in the direction of the incident particle. i,e $\theta_2=0$. Then from equation (1) & (2) we get

$$m_1u_1= m_1v_1\cos\theta_1 + m_2v_2 \quad \& \quad 0= m_1v_1\sin\theta_1$$

(iii) Elastic collision of two identical particles :- the identical particles moves at right angle to each other after the collision.

Some important points about the collision

(i) The total energy and total momentum remains conserved in elastic as well as inelastic collision.

(ii) The kinetic energy of the particles remains conserved before and after the collision but at the instant of collision the kinetic energy does not remains conserved.

| Collision | Kinetic energy | Coefficient of restitution | Main domain |
|---|---|---|---|
| **Elastic** | conserved | e=1 | Between atomic particles |
| **Inelastic** | Not conserved | 0<e<1 | Between ordinary objects |
| **Perfectly inelastic** | Max. lose of K.E | e=0 | During shooting |
| **Super elastic** | K.E increases | e>1 | In explosions |

## SOME IMPORTANT MCQ

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Q. 1 | Which of the following is NOT a correct unit for work? | | | | | | | | |
| | A. erg | B. Joule | C. watt | D. Newton· meter | | | | | |
| Q. 2 | Which of the following groups does NOT contain a scalar quantity? | | | | | | | | |
| | A. velocity, force, power | | | B. displacement, acceleration, force | | | | | |
| | C. acceleration, speed, work | | | D. energy, work, distance | | | ans: B | | |
| Q.3 | 30. Which of the following bodies has the largest kinetic energy? | | | | | | | | |
| | A. Mass 3M and speed V | | | B. Mass 3M and speed 2V | | | | | |
| | C. Mass 2M and speed 3V | | | D. Mass M and speed 4V | | | ans: C | | |
| Q.4 | The weight of an object on the moon is one-sixth of its weight on Earth. The ratio of the | | | | | | | | |
| | kinetic energy of a body on Earth moving with speed V to that of the same body moving with | | | | | | | | |
| | speed V on the moon is: | | | | | | | | |
| | A. 6:1 | B. 36:1 | C. 1:1 | D. 1:6 | ans: C | | | | |
| Q.5 | The amount of work required to stop a moving object is equal to: | | | | | | | | |
| | A. the velocity of the object | | | B. the kinetic energy of the object | | | | | |
| | C. the mass of the object times its acceleration | | | | | | | | |
| | D. the mass of the object times its velocity | | | | | ans: B | | | |
| Q.6 | A 5.0-kg cart is moving horizontally at 6.0m/s. In order to change its speed to 10.0m/s, the | | | | | | | | |
| | net work done on the cart must be: | | | | | | | | |
| | A. 40 J | B. 90 J | C. 160 J | D. 400 J | ans: C | | | | |
| Q.7 | A watt second is a unit of: | | | | | | | | |
| | A. force | E. energy | C. displacement | | B. power | | | | |
| Q.8 | A kilowatt-hour is a unit of: | | | | | | | | |
| | A. power | B. energy/time | | C. work | D. power/time | | ans: C | | |
| Q.9 | A force on a particle is conservative if: | | | | | | | | |
| | A. its work equals the change in the kinetic energy of the particle | | | | | | | | |
| | B. it obeys Newton's second law | | | | C. it obeys Newton's third law | | | ans: D | |
| | D. its work depends on the end points of every motion, not on the path between | | | | | | | | |
| Q.10 | No kinetic energy is possessed by | | | | | | | | |
| | A. a shooting star | | B. a rotating propeller on a moving airplane | | | | | | |
| | C. a pendulum at the bottom of its swing | | | | | | | | |
| | D. an elevator standing at the fifth floor | | | | | | | | |