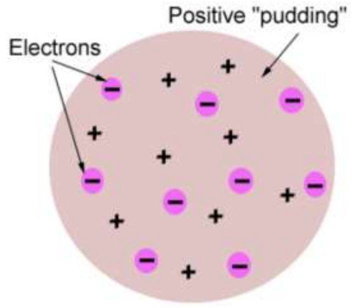
Unit-8 (A) Atom and Nucleus

1 J J Thomson's Model of an Atom:

According to JJ Thomson model of an atom, an atom is just like a watermelon in which seeds behave as electrons and reddish parts behave as $+vely$ charged matter. Atom as a whole is electrically neutral.



Failure of JJ Thomson Model:

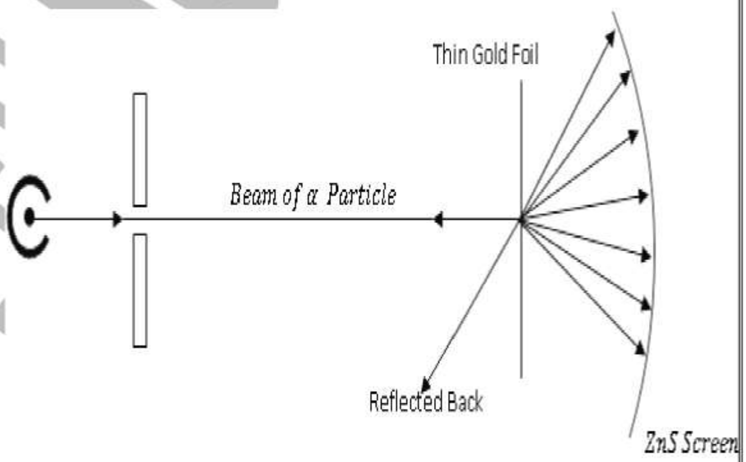
- *It could not explain the origin of spectral series of hydrogen atom.*
- *It fails to explain the large angle scattering of hydrogen atom.*

2 Rutherford α-particle Scattering Experiment [imp]:

Rutherford along with Geiger and Marsden performed a number of experiments on the scattering of α-particle due to atoms of thin gold foils. The experimental demonstration of their model is as below.

Experimental Setup:

A narrow beam of α particle obtained from a radioactive substance placed in a lead cavity is generated, which is allowed to fall on a thin gold foil, after passing through a narrow slit. After striking the gold foil, the beam is allowed to fall on ZnS screen and the pattern is seem with the help of movable detector. As shown in fig.



Observation:

Rutherford observed that

1. Most of the α-particle passed through the gold foil un-deflected.
2. Some of α-particle was deflected through small angle.
3. A few (about 1in 8000) regained their path i.e. deflected at 180° angle.

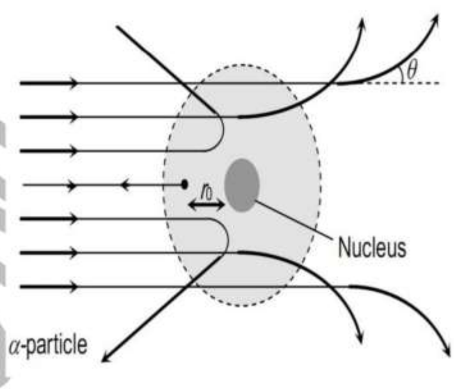
Conclusion:

1. As most of α-particle passed un-deflected, which indicate that there is large empty space inside the atom
2. As α-particle is a $+vely$ charge particle, it can deflect only by large $+vely$ charge particle it means whole $+ve$ charge of an atom lies at a single point which was called nucleus.

3. Using high speed α-particle we can find out the size of nucleus of order of $10^{-14}m$.
4. The nucleus is surrounded by electrons having equal $-ve$ charge to $+ve$ charge of nucleus. Atom as a whole is electrically neutral.

3 Distance of Closest Approach: Estimation of Nuclear Size imp:

In Rutherford α-ray scattering experiment the α-particle stops for a moment and then retraces its path. Here the distance r_0 is called the distance of closet approach.



At this distance the entire K.E of α-particle gets convert into electrostatic potential energy. Here the kinetic energy of α-particle is $K_a = \frac{1}{2}mv^2$ and electrostatic P.E. of α-particle and nucleus is

$$U = K\frac{q_1 q_2}{r_0} = K\frac{2e\cdot ze}{r_0}$$

Now $$\frac{1}{2}mv^2 = \frac{K\cdot 2ze^2}{r^0}$$

$\Rightarrow$ $$r_0 = \frac{2Kze^2}{k_\alpha} = \frac{4Kze^2}{mv^2}$$

Clearly the radius of the nucleus must be small then r_0.

In Rutherford experiment energy of α particle was 5.5MeV.

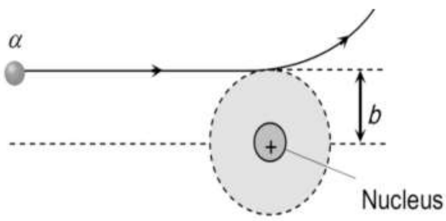
So $$K_\alpha = 5.5MeV = 5.5 \times 1.6 \times 10^{-13}J$$

Also $z = 79$So $$r_0 = \frac{2\times 9\times 10^9 \times 79 \times (1.6\times 10^{-19})^2}{5.5\times 10^{-13}} = 4.13\times 10^{-14}m$$

So we can say that radius of the nucleus is of order of $10^{-15}m$.

4 Impact Parameter imp:

It is defined as the perpendicular distance of the velocity vector of α-particle from centre of the nucleus when the path of α-particle is un-deflected.



Rutherford deduced a relation between impact parameter and scattering angle θ.

As $$b = \frac{1}{4\pi E_0}\frac{ze^2 cot\frac{\theta}{2}}{\frac{1}{2}mv^2} \quad \text{Or} \quad b = \frac{Kze^2 cot\frac{\theta}{2}}{E}$$

- If impact parameter is large then repulsive force experienced by α particle is small and vice versa.
- For a head on collision the impact parameter $b = 0\ as\ \theta = 180° \Rightarrow cot90 = 0$

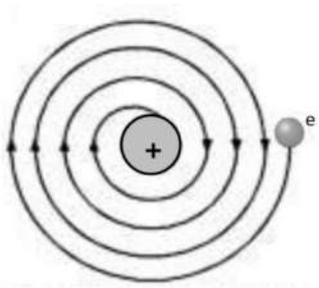
5 Rutherford Model of an Atom imp

On the basis α-ray scattering experiment Rutherford proposed a model of atom. The main points are:

1. Most of the mass and all $+ve$ charge of an atom are concentrated at centre of the atom called nucleus.
2. The size of the nucleus $(10^{-15}m)$ is extremely small as compared to size of the atom $(10^{-10}m)$. So most of the space in a atom is empty.
3. The number of electrons in a atom is equal to $+ve$ charge in the nucleus hence atom is electrically neutral.
4. Electron revolves around the nucleus in circular orbit the centripetal force required to revolve electron is provide by the columbium force of attraction between nucleus and electrons.

6 Limitations:

1. **It fails to Explains the Stability of an Atom:**



As according to Rutherford model of an atom electron revolve around the nucleus in circular orbits, but, but according to electromagnetic theory a acceleration charge particle must radiate energy. By doing this the path of e^- will become shorter and spiral and ultimately it will fall into the nucleus.

(ii) In Rutherford model electron may revolve in orbits of any possible radii so it should emit continuous spectra. But an atom like H emits discrete spectra. Thus Rutherford model cannot explain spectra of H-atom.

7 Bohr's Model of an Atom m.imp:

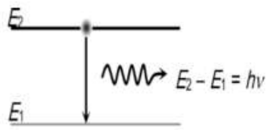
To explain stability and spectrum of an atom, Bohr applied Planck's quantum theory of radiations to Rutherford model of atom. The main postulates of Bohr model are as given below:

1) An atom has a small $+vely$ charged core where whole of mass of an atom is supposed to be concentrated. This core is called nucleus of the atom.

2) The electron revolves around the nucleus in definite energy orbits and does not radiate any energy during their rotation.

3) The electron can revolve only in those orbits, in which its angular momentum is an integral multiple of $h/_{2\pi}$ i.e $mvr = \frac{nh}{2\pi}$

4) Electron radiates or absorbs energy when it moves from one orbit to another orbit.



If a electron in one orbit absorb $h\nu$ amount of energy then it jumps to higher energy orbits. i.e $E_2 = E_1 + h\nu$

Or $h\nu = E_2 - E_1$

8 Bohr's Theory of H-Atom:

(1) Radius of H-atom

As H-atom have nucleus charge $+e$ and electronic charge $-e$. Then coulomb's force of attraction between the nucleus and the electron is given by $F = \frac{Ke^2}{r^2}$

This coulomb ion force provides the necessary centripetal force to the electron to revolve in circular orbits around the nucleus. i.e $\frac{mv^2}{r} = \frac{Ke^2}{r^2}$

or $$mv^2 = \frac{Ke^2}{r} \dots\dots\dots (i)$$

Now according to Bohr's postulate of H-atom $mvr = \frac{nh}{2\pi}$ $\Rightarrow$ $v = \frac{nh}{2\pi mr} \dots\dots\dots\dots (ii)$

Using in eq. (i) we get $$m\left(\frac{nh}{2\pi mr}\right)^2 = \frac{Ke^2}{r}$$

$$= \frac{mn^2h^2}{4\pi^2m^2r^2} = \frac{Ke^2}{r} \quad \text{or} \quad \frac{n^2h^2}{4\pi^2mr} = Ke^2$$

$$\Rightarrow \quad r = \frac{n^2h^2}{4\pi^2Kme^2} \dots\dots\dots\dots\dots\dots\dots (iii)$$

But $$\frac{h^2}{4\pi^2Kme^2} = constant \quad \Rightarrow \quad r \ \alpha \ n^2$$

Thus radius of an orbit is directly proportional to the square of principle quantum number of the orbit.

The radius of the inner most orbits $(n = 1)$ in the hydrogen atom is called Bohr's radius (a_0)

as $$a_0 = \frac{h^2}{4\pi^2me^2K} = \frac{(6.6\times10^{-34})^2}{4\times(3.14)^2\times9.1\times10^{-31}\times9\times10^9\times(1.6\times10^{-19})^2} = 5.29\times10^{-11}m \simeq 0.53A^\circ$$

(ii) Speed of Electron in an Orbit:

As we know from eq. (ii) that $$v = \frac{nh}{2\pi mr}$$

Using eq. (iii) we get $$v = \frac{nh}{2\pi m}\left(\frac{4\pi^2Kme^2}{n^2h^2}\right) = \frac{2\pi Ke^2}{nh} \dots\dots\dots\dots\dots\dots\dots\dots\dots (iv)$$

Or $v \ \alpha \ \frac{1}{n}$ where $\frac{2\pi Ke^2}{h}$ is a constant

Thus speed of electron in an orbit is inversely proportional to the principle quantum number.

(iii) Energy of a electron in an orbit:

The total energy of an electron in an orbit is equal to the sum of K.E and P.E, i.e

$$E = K.E + P.E \dots\dots\dots\dots\dots (v)$$

From eq. (i) we get $K.E = \frac{Ke^2}{2r}$

And P.E=Potential of e^- in a orbit × charge on electron $= \frac{-Ke^2}{r}$

Using in eq. (v) we get $E = \frac{1}{2}\frac{Ke^2}{r} - \frac{Ke^2}{r} = -\frac{1}{2}\frac{Ke^2}{r} = -\frac{1}{2}Ke^2\left(\frac{4\pi^2 Kme^2}{n^2h^2}\right) = \frac{-2\pi^2K^2me^4}{n^2h^2}$

Multiplying and dividing R.H.S by ch we get $E = \left(\frac{2\pi^2K^2me^4}{ch^3}\right)\frac{ch}{n^2} = -\frac{R_h ch}{n^2}$

Where $R_h = \frac{2\pi^2K^2me^4}{ch^3} = 1.09 \times 10^7 m^{-1} = Rydberge\ Constant$

Or $E \ \alpha\ -\frac{1}{n^2}$

Thus total energy is inversely proportion to square of principle quantum number.

Here – ve sign indicate that the total energy of e^- in an orbit is due to attractive force between e^- and nucleus.

9 Spectral Series of H-Atom:

Before studying spectral series of H-atom we will study absorption and emission spectra.

(A) Emission Spectra:

When an electron jumps from a higher energy level to lower energy level, then it emits energy in form of radiations of certain wavelength called emission spectra.

The amount of energy emitted by electron is $\boldsymbol{hv = E_2 - E_1}$.

(B) Absorption Spectra:

When an element is heated to a high temperature then it absorb energy and emit certain spectra called absorption spectra. In this case electron absorb $\boldsymbol{hv}$ amount of energy and excite to higher energy level

as $\boldsymbol{E_1 + hv = E_2}$

10 Spectral Series of H-atom imp:

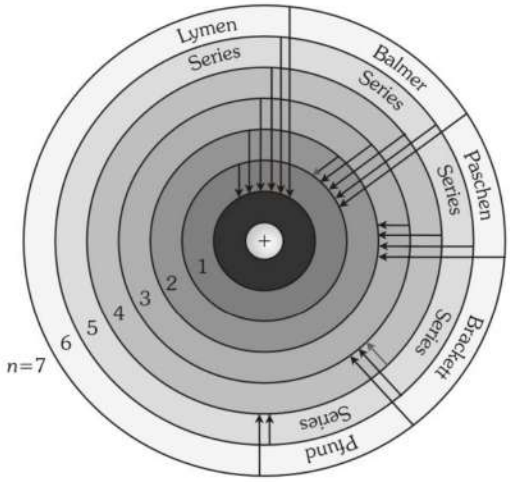
As electron move from higher energy orbit to lower energy orbit then it emits line spectra of frequency v

as $$hv = E_{n_2} - E_{n_1} = \frac{2\pi^2 mK^2e^4}{h^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

or $$v = \frac{2\pi^2 mK^2e^4}{h^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

or $$\bar{v} = \frac{1}{\lambda} = \frac{v}{c} = \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]\frac{2\pi^2 mK^2e^4}{ch^3}$$

or $$\bar{v} = R_h\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$



Where R_h is called Rydberg constant.

Now the origin of the various spectral series in the H-atom can be explained as now.

i. Lyman Series:

If an electron energy level to $n = 1$ then electron emits Lyman series in ultraviolet region.

This series is given by $\bar{v} = \frac{1}{\lambda} = R\left[\frac{1}{(1)^2} - \frac{1}{(n_2)^2}\right]$ Where $n_2 = 2,3,4 \dots\dots\dots\dots\dots\dots\dots$

ii. Balmer Series:

If a electron jumps from higher energy level to $n = 2$ level then electron emits Balmer Series I visible region. This series is given by $\bar{v} = \frac{1}{\lambda} = R\left[\frac{1}{(2)^2} - \frac{1}{(n_2)^2}\right]$ Where $n_2 = 3,4,5 \dots\dots\dots\dots\dots\dots\dots$

iii. Paschen Series:

If an electron jumps from higher energy level to n=3 level then electron emits Paschen series in infrared region. This series is given by $\bar{v} = \frac{1}{\lambda} = R\left[\frac{1}{(3)^2} - \frac{1}{(n_2)^2}\right]$ Where $n_2 = 4,5,6 \dots\dots\dots\dots\dots\dots\dots$

iv. Brakett Series:

If a electron jumps from higher energy level to $n = 4$ level then it emits Brakett series in infrared region. This series is given by $\bar{v} = \frac{1}{\lambda} = R\left[\frac{1}{(4)^2} - \frac{1}{n_2^2}\right]$ Where $n_2 = 5,6,7 \dots\dots\dots\dots\dots\dots\dots$

v. P fund Series:

If an electron jumps from higher energy level to n=5 level then it emits P fund series in infrared region. This series is given by

$$\bar{v} = \frac{1}{\lambda} = R\left[\frac{1}{(5)^2} - \frac{1}{n_2^2}\right] \quad \text{Where } n_2 = 6,7,8 \dots\dots\dots\dots\dots\dots\dots$$

Bohr explained successfully the series of Lyman, Balmer and Paschen series and predicted Brackett and P fund series which were latterly explained by Brakett and P fund.

Q. Find the wavelength of the electron orbiting in the first excited state in hydrogen atom.

Ans. The energy of nth state of the hydrogen atom is given as; $E_n = -\frac{13.6}{n^2}$

So, in first excited state, n = 2 Therefore: $E_2 = \frac{-13\ 6.eV}{4} = -3.4\ eV$

So $\lambda = \frac{hc}{E} = \frac{6.63\times10^{-34}\times3\times10^{8}}{3.4\times1.6\times10^{-19}} = 3.656\times10^{-7}\,m$

11 Energy Level Diagram For H-atom imp:

It is a diagram in which the energies of different stationary states of an atom are represented by parallel horizontal lines, drawn according to some suitable energy scale.

Energy level for H-atom:

According to Bohr's theory, the total energy of electron in nth orbit is

$$E_n = -\frac{2\pi^2 mK^2e^4}{n^2h^2} = -\frac{2\times(3.14)^2\times(9.1\times10^{-31})\times(9\times10^{9})^2(1.6\times10^{-19})^4}{(6.67\times10^{-34})^2n^2} - \frac{13.6}{n^2}eV$$

Or $$E_n = -\frac{13.6}{n^2}eV$$

Now energy of electron in first orbit $(n = 1)$ is $E_1 = \frac{-13.6}{(1)^2} = -13.6\ eV$

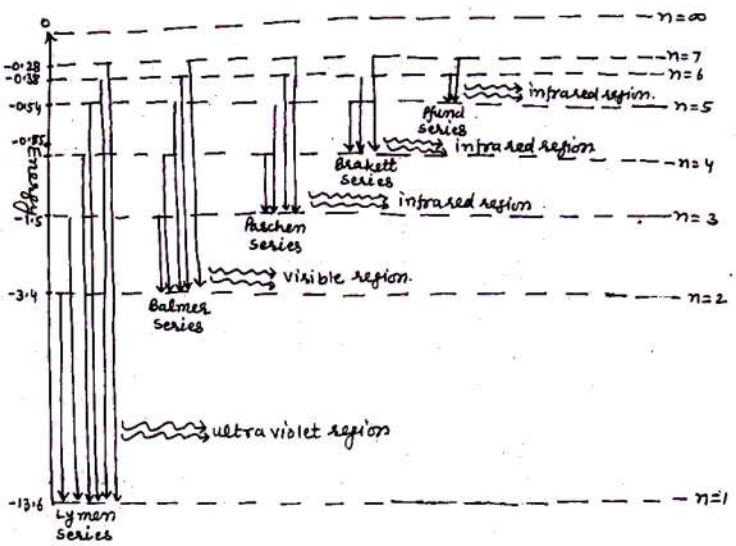
Again energy of electron in second orbit $(n = 2)$ is $E_2 = \frac{-13.6}{4} = -3.4\ eV$

Again for $n = 3$ $E_3 = \frac{-13.6}{9} = -1.51eV$

And $E_4 = \frac{-13.6}{16} = -0.85$

$E_5 = \frac{-13.6}{25} = -0.54eV$

And $E_6 = \frac{-13.6}{(6)^2} = 0.37eV$



Clearly an electron has certain definite energy while revolving in a orbit. This is called quantization of energy of electron in a atom.

In case of H-atom energy state of maximum energy $E_1 = -13.6$ is called ground state and other $E_2, E_3, \dots\dots\dots\dots E_n$ are called excited states.

12 Limitations of Bohr's Theory:

- Bohr's theory explained certain spectral lines of H-atom but it fails to explain fine features of Hydrogen spectrum.
- It could not explain the spectral series of other atoms except Hydrogen.
- According to Bohr Theory, the orbits of electrons are circular only but elliptical orbits are also possible.
- As electron also have wave nature so exact position of electron cannot be explained by Bohr's Theory.
- It does not tell anything about the relative intensities of spectral series.
- It does not explain the further splitting of spectral series in magnetic field (Zeeman Effect) or in analectic field (Stark effect).

13 Excitation and Ionization Potential:

Excitation Energy:

The amount of energy required by an electron to jump from ground state to any other energy state is called excitation energy.

First excitation energy of $H = E_2 - E_1 = -3.4 - (-13.6) \ = 10.2\ eV$

Second excitation energy of $H = E_3 - E_1 = -1.51 - (-13.6) = 12.09\ eV$

Ionization Energy:

The amount of energy required to remove an election from an isolated gasses atom is called ionization energy.

Ionization energy of $H = E_\infty - E_1 = 0 — 13.6 = 13.6\ eV$

Excitation Potential:

The amount of potential require to jump a electron from lower energy level to higher energy level is called excitation potential.

First excitation potential of $H - atom = -3.4 - (-13.6) = 10.2\ V$

Second excitation potential of $H - atom = -1.51 — 13.6 = 12.09\ V$

Ionization potential:

The amount of potential require to just eject a electron from isolated gases atom is called ionization potential. Ionization potential of $H - atom = 0 - (-13.6) = 13.6\ V$